Rationality of Expectations: Another OCA Criterion?
A DSGE Analysis

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Abstract

The Walters critique of EMU presumed that pro-cyclical country-specific real interest rates would incorporate significant macroeconomic instability in an environment of asymmetric shocks. The literature on optimum currency areas suggests a number of criteria to minimize this risk, such as market flexibility, high degrees of openness, financial integration or similarity in inflation rates. In this paper, we argue that an essential part of macroeconomic volatility in a monetary union's member country also depends on the mechanism of forming expectations. This is mainly due to (i) the construction of *ex ante* country-specific real interest rate, implying a strong or weak negative correlation with current inflation rate and (ii) anticipated (and hence smoothed) loss in competitiveness and boom-bust cycle. In a 2-region 2-sector New Keynesian DSGE model, we apply 5 different specifications of *ex ante* real interest rates, based on commonly considered types of expectations: rational, adaptive, static, extrapolative and regressive, as well as their hybrids. Our simulations show that rational expectations dominate the other specifications in terms of minimizing the volatility of the most macroeconomic variables. This conclusion is generally insensitive to which group of agents (producers or consumers) and which region (home or foreign) forms the expectations. It also turns out that for some types of expectations the Walters critique indeed applies, i.e. the system does not fulfil the Blanchard-Kahn conditions or the system's companion matrix has explosive eigenvalues.

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1 Introduction

The process of absorbing asymmetric shocks in monetary unions has been investigated in a long strand of literature, dating back to classical papers by Mundell (1961), McKinnon (1963) and Kenen (1969). They laid foundations for the optimum currency area (OCA) literature and were followed by a large number of theoretical and empirical contributions (see Mongelli, 2002, for an overview). This literature enumerates the conditions ensuring that occurance of asymmetric shocks should ether be unlikely or at least followed by a smooth adjustment process. The list includes i.a. price and wage flexibility, production factor mobility, openness to trade (preferably concentrated within the monetary union), financial market integration, fiscal and political integration, diversification of production and consumption structures, as well as similarity of inflation rates.

The market-based adjustment involves two phenomena of particular interest. Firstly, if a positive shock leads to an expansion in a small economy and an increase in inflation rate, the real interest rates fall, which additionally fuels the boom. This procyclical feature of country-level real interest rates (see i.a. European Commission, 2006; Roubini, Parisi-Capone, Menegatti, 2007; European Commission, 2008; Torój, 2009a) was first emphasized by Walters (1994) as a source of inherent instability, potentially leading to a break-up of a heterogenous monetary union. Secondly, the "Walters critique" does not account for the fact that a boom in a small open economy erodes the competitiveness of domestic products. Consequently, this leads to a fall in demand and realignment to equilibrium. This competitiveness channel (see i.a. European Commission, 2006; Torój, 2009a) is effective as long as flexible markets ensure timely real appreciation in the domestic economy and the level of production remains sufficiently sensitive to the real exchange rate against the rest of the monetary union. A symmetric reasoning applies for an adverse shock (followed by an increase in real interest rate and real exchange rate depreciation).

The determinants of adjustment dynamics were already described in the initial phases of OCA literature, including pioneering phase of 1960s and reconciliation phase of 1970s (see Mongelli, 2002). However, since then, the analytical frameworks in macroeconomics advanced substantially. In particular, the rational expectations revolution (originating from seminal contributions by Muth, 1961 and Lucas, 1976) introduced dynamic stochastic general equilibrium models as mainstream modelling devices. While the New Keyesian, expectation-based models were developed, the OCA literature – as evaluated by Mongelli (2002) – progressed relatively slowly in a "reassessment phase" in 1980 due to unclear monetary integration prospects of that time, followed by an empirical phase in 1990s when EMU was already in preparation. However, to the best of our knowledge, the model-based work on OCA criteria does not explicitly analyse the role of expectations in the realignment of a small economy after an asymmetric shock.

In this paper, we attempt to fill this gap by a formalized discussion of the role that expectations play in the adjustment process. We ask whether the choice of a particular

mechanism of forming expectations could significantly influence a country's capacity to absorb asymmetric shocks, and – if yes – which mechanism seems to be the most efficient. To address this issue, we apply a DSGE model of a 2-region monetary union and investigate impulse-response functions and variances generated by this model under different expectation types. Also, we check whether the Walters critique applies for some particular types, i.e. whether the model's stability conditions hold. Finally, we allow consumers and producers (as well as home and domestic agents) to differ in terms of expectation types and ask whether some of the combinations yield lower volatility of macroeconomic variables than under perfect homogeneity.

The rest of the paper is organized as follows. Section 2 reviews the existing literature. In Section 3, we develop a New Keynesian model of a monetary union that will be used as an analytical tool to answer the above questions. In Section 4 we present the calibration, considered expectation types and simulation results. Section 5 concludes.

2 Expectations and OCA: literature review

In a monetary union, agents could anticipate a number of strong links between the home and foreign economy being at work (see also Torój, 2009b). Firstly, the common central bank reacts to foreign demand shocks with a move in the common policy rate, which in turn translates directly into a change in domestic monetary conditions. Secondly, a foreign shock affects future price dynamics abroad. As a result, the real exchange rate and domestic monetary conditions change. Thirdly, foreign business cycle affects the domestic output due to international trade and investment links. Economic agents are therefore capable to predict an economic slowdown at home when they observe one in other countries.

Outside a monetary union, the forecasting capacity of agents is more limited as the nominal exchange rate fluctuations and separate monetary policies of domestic and foreign central banks introduce additional degrees of freedom.

The ample literature on the endogeneity of OCA criteria (e.g. integration of finance and trade in a common currency area, see e.g. Frankel and Rose, 1998 or European Commission, 2008) suggests growing interdependence of individual countries' output gaps after creating a monetary union. As a result, the forward-looking behaviour developed by agents might equip the economy with a strong stabilizing force inside the monetary union.

Firstly, the adjustment via competitiveness channel could be anticipated and help in containing the preceding boom in advance. For example Andersson, Gieseck, Pierluigi, Vidalis (2008) note that this forward-looking behaviour has been institutionalized in Belgian enterprises. The growth rate of wages within the next 2 years cannot exceed predicted growth rate of wages in Germany, France and the Netherlands – Belgium's main trading partners. Also, Calmfors and Johansson (2006) show that an irrevocably fixed exchange rate might prevent exporters from signing long-term wage contracts. Secondly, the process of forming expectations influences the definition of *ex ante* real

interest rate and hence the dynamics of the real interest rate effect. The type of expectations decides how strongly inflation expectations are correlated with the current inflation rate. If this correlation was weak, a boom in economy would not translate into growing expectations, at least not in the short run. European Commission (2006) emphasizes that there is more cross-country variation in inflation rates than in the measures of inflation expectations in euro area countries. They also stress that producers and consumers might differ significantly as regards the mechanisms of forming inflation expectations. Namely, producers should put more weight on external price dynamics, be more forward-looking and more rational.

The degree of forward-lookingness as a factor of adjustment efficiency is considered in a related study by Allsopp and Vines (2010). They introduce a differentiation between forward- and backward-looking pricesetters, as well as consumers who do and do not smooth their consumption over time due to partly limited access to the credit markets. The prevalence of backward-looking behaviour leads in principle to model instability as a result of Walters critique validity.

Finally, apart from asymmetric shock absorption, the literature describes other real aspects of consumers' expectations and perceptions in the aftermath of euro adoption. They include i.a. the "euro illusion", i.e. an over-proportional growth of perceived inflation rate (see National Bank of Poland, 2009, for an extensive overview). It might result in rising inflation expectations, second-round effects and drop in consumption due to lower perceived real income.

3   New Keynesian model of a monetary union

In this Section, we develop a DSGE model for the analysis of country-level adjustment to shocks under different expectation types. It builds strongly upon multi-region currency union models with possible heterogeneity, such as e.g. ones considered in the works by Benigno (2004), Lombardo (2006), Brissimis and Skotida (2008), Blessing (2008) or Kolasa (2009). The currency union consists of 2 regions. The whole economy of the monetary union is represented by the interval $\langle 0; 1 \rangle$, whereby the first region (say, home economy) is indexed over $\langle 0; w \rangle$ (relative size of the region: w), and the second (foreign economy) is indexed over $\langle w; 1 \rangle$.

As the behaviour of the nontradable sector is considered to be a crucial element of adjustment dynamics (see e.g. European Commission, 2008, 2009), both economies consist of two sectors. Each of them is characterized by price rigidities, modelled with Calvo (1983) mechanism. Conventionally, consumers in each region maximize their utility and producers in each sector – their present and discounted future profits. International exchange of goods incorporates the competitiveness channel of adjustment into the model and ensures that in the long run both economies return to their equilibrium after a shock. This is also true for a small economy that does not have an autonomous monetary policy, which is modelled for the entire currency union via a simple Taylor rule with smoothing.

The model incorporates a number of standard New Keynesian nominal and real rigidities, such as price stickiness modelled with the Calvo mechanism, wage stickiness, price and wage indexation or consumption habits. While monetary policy is always symmetric (with a possibly asymmetric transmission mechanism though), there are four other shocks in the model that can be asymmetric (region-specific): demand shocks, supply shocks in the tradable and non-tradable sector, as well as labour supply shocks. Henceforth, parameters of the foreign economy are denoted analogously to home economy and marked with an asterisk, e.g. σ and σ^*. Lowercase letters denote the log-deviations of their uppercase counterparts from the steady-state values. More detailed technical exposition of the model derivation can be found in the Appendix.

3.1 Household decisions

Intratemporal allocation of consumption

Households get utility from consumption and disutility from hours worked. In addition, utility from consumption depends on consumption habits formed in the previous period (see Smets and Wouters, 2003; Kolasa, 2009). The constant relative returns to scale utility function takes the following form (compare Galí, 2008):

$$U_t\left(C_t, N_t, H_t\right) = \varepsilon_{d,t} \frac{\left(C_t - H_t\right)^{1-\sigma}}{1-\sigma} - \varepsilon_{l,t} \frac{N_t^{1+\phi}}{1+\phi} \tag{1}$$

where C_t – consumption at t, H_t – stock of consumption habits at t, N_t – hours worked at t, $\sigma > 0$ and $\phi > 0$. Consumption habits are assumed to be proportional to consumption at $t-1$ (see Fuhrer, 2000; Smets and Wouters, 2003):

$$H_t = hC_{t-1} \tag{2}$$

with $h \in [0; 1)$ The overall consumption index aggregates the tradable and nontradable consumption bundles:

$$C_t \equiv \left[(1-\kappa)^{\frac{1}{\delta}} C_{T,t}^{\frac{\delta-1}{\delta}} + \kappa^{\frac{1}{\delta}} C_{N,t}^{\frac{\delta-1}{\delta}}\right]^{\frac{\delta}{\delta-1}} \tag{3}$$

where $\kappa \in (0; 1)$ characterizes the share of nontradables in the home economy and $\delta > 0$ is the elasticity of substitution between the goods produced in both sectors.
The domestic consumption of tradables at t consists of goods produced at home, $C_{H,t}$, and abroad, $C_{F,t}$:

$$C_{T,t} \equiv \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{4}$$

An analogous relationship holds for the foreign economy. Given this, α is an intuitive measure of degree of openness and $1 - \alpha$ – home bias in consumption; $\eta > 0$ is the elasticity of substitution between home and foreign tradables.

The consumption of domestic tradable goods in the home economy $(C_{H,t})$ and in the foreign one $(C^*_{H,t})$ is defined as:

$$C_{H,t} \equiv \left[\left(\tfrac{1}{w}\right)^{\frac{1}{\varepsilon_T}} \int_0^1 \left(\int_0^w C^j_{H,t,k} dj \right)^{\frac{\varepsilon_T-1}{\varepsilon_T}} dk \right]^{\frac{\varepsilon_T}{\varepsilon_T-1}}$$
$$C^*_{H,t} \equiv \left[\left(\tfrac{1}{w}\right)^{\frac{1}{\varepsilon_T}} \int_0^1 \left(\int_0^w C^{j*}_{H,t,k} dj \right)^{\frac{\varepsilon_T-1}{\varepsilon_T}} dk \right]^{\frac{\varepsilon_T}{\varepsilon_T-1}} \tag{5}$$

The parameter $\varepsilon_T > 1$ measures the elasticity of substitution between various types of goods in international trade, k indexes the variety of goods, and j – the households (integral over j reflects the difference in both economies' size).

The nontradable consumption bundles, domestic $(C_{N,t})$ and foreign $(C_{N*,t})$, are characterized in a similar fashion as:

$$C_{N,t} \equiv \left[\left(\frac{1}{w}\right)^{\frac{1}{\varepsilon_N}} \int_0^1 \left(\int_0^w C^j_{N,t,k} dj \right)^{\frac{\varepsilon_N-1}{\varepsilon_N}} dk \right]^{\frac{\varepsilon_N}{\varepsilon_N-1}}$$

$$C_{N*,t} \equiv \left[\left(\frac{1}{1-w}\right)^{\frac{1}{\varepsilon_{N*}}} \int_0^1 \left(\int_w^1 C^{j*}_{N*,t,k} dj \right)^{\frac{\varepsilon_{N*}-1}{\varepsilon_{N*}}} dk \right]^{\frac{\varepsilon_{N*}}{\varepsilon_{N*}-1}}$$

Consequently, ε^N and ε^{N*} is defined as elasticity of substitution between various types of nontradable goods.

Households maximize at t the discounted flow of future utilities:

$$E_t \sum_t^\infty \beta^t U \left(C_t, N_t, H_t \right) \rightarrow \max_{C,N} \tag{6}$$

where $\beta \in (0,1)$ is households' discount factor. Maximization of (32) is subject to a sequence of standard period budget constraints faced by a representative household. It leads to the standard first order conditions that define the demand for various types of goods as a declining function of their relative prices and the demand for the bundle to which this good belongs.

Intertemporal allocation of consumption

The standard condition of intertemporal optimality, i.e. equality between marginal loss in utility due to buying a security at t instead of allocating this money to consumption and the discounted payoff at $t+1$, also expressed in terms of marginal growth of future utility, lead to the following log-linearized dependence (Euler equation for

consumption):

$$c_t = \frac{h}{1+h}c_{t-1} + \frac{1}{1+h}E_t c_{t+1} - \frac{1-h}{(1+h)\sigma}(i_t - E_t \pi_{t+1} - \rho) + \\ + \frac{1-h}{(1+h)\sigma}(\varepsilon_{d,t} - E_t \varepsilon_{d,t+1}) \tag{7}$$

where i_t denotes short-term nominal interest rate at t, $E_t \pi_{t+1}$ – expected domestic consumer price growth, $\rho = -ln\beta$ – natural interest rate corresponding to the households' discount factor β.

3.2 International prices

Define bilateral terms of trade between the home and foreign economy as:

$$S_t \equiv \frac{P_{H,t}}{P_{F,t}} \tag{8}$$

Also, define internal terms of trade as price ratio between tradables and nontradables:

$$X_t \equiv \frac{P_{T,t}}{P_{N,t}} \tag{9}$$

3.3 International risk sharing

Household can smooth their consumption not only in time, but also in international financial markets (Blessing, 2008; Galí, 2008; Kolasa, 2009; Lipińska, 2008). Under complete markets, equation (53) holds for both home and foreign economy. This allows to derive the following log-linearized relation between home and foreign consumption and the real exchange rate q_t (see also Chari, Kehoe, McGrattan 2002):

$$\frac{\sigma}{1-h}(c_t - hc_{t-1}) - \varepsilon_{d,t} = \frac{\sigma^*}{1-h^*}(c_t^* - h^* c_{t-1}^*) - \varepsilon_{d,t}^* - q_t \tag{10}$$

3.4 Producers

Real marginal costs

The producers of variety k in the tradable or nontradable bundle face a single-factor production function with constant returns to scale (see Galí, 2008). Following Clarida, Galí, Gertler (1999), we assume away the price deviations of individual varieties within a sector as of second-order importance in the proximity of the steady state.

The real marginal cost (as log-deviation from the steady-state) is calculated as a difference between the wage level in the region (w_t) and the sectoral producer price log-level plus the log of marginal labour product (mpn) (see Galí and Monacelli, 2005), which can be expressed in both sectors as:

$$mc_t^H = (w_t - p_t) - \alpha s_t - \kappa x_t - (a_t^H + \varepsilon_t^H) \tag{11}$$

$$mc_t^N = (w_t - p_t) + (1 - \kappa) x_t - \left(a_t^N + \varepsilon_t^N\right) \tag{12}$$

with supply shocks in both sectors denoted as ε_t^H and ε_t^N respectively. a_t^H and a_t^N are log labour productivities.

Pricing decisions

There are nominal price rigidities in the economy. Following the usual approach in the New Keynesian literature, we model them by means of the Calvo (1983) scheme. In a given period, a fraction θ of producers are not allowed to reoptimize their prices in reaction to economic innovations and must sell at the price from the previous period. The probability of being allowed to reoptimize the price is equal across producers: $1 - \theta$ in each period, independently of the amount of time elapsed since the last price change.

Some of the producers (fraction ω of reoptimizers) allowed to change their price do not really reoptimize. Following Galí and Gertler (1999) we assume that the change in price is partly implemented as an indexation to past inflation. This mechanism leads to a hybrid Phillips curve (see Galí and Gertler, 1999; Galí, Gertler, López-Salido 2001), commonly considered to outperform the purely forward-looking specifications in terms of empirical goodness-of-fit. Inflation is modelled separately in the tradable and nontradable sector.

The abovementioned assumptions lead to the following hybrid Phillips curve in the H sector:

$$
\begin{aligned}
\pi_t^H = {} & \frac{\omega^H}{\theta^H + \omega^H \left[1 - \theta^H \left(1 - \beta\right)\right]} \pi_{t-1}^H + \frac{\beta\theta^H}{\theta^H + \omega^H \left[1 - \theta^H \left(1 - \beta\right)\right]} E_t \pi_{t+1}^H + \\
& \frac{\left(1 - \omega^H\right)\left(1 - \theta^H\right)\left(1 - \beta\theta^H\right)}{\theta^H + \omega^H \left[1 - \theta^H \left(1 - \beta\right)\right]} mc_t^H
\end{aligned}
\tag{13}
$$

and analogously for N.

3.5 Market clearing conditions

Equilibrium on the world markets of individual goods requires equality of overall production and consumption of every variety k in the basket of domestically produced tradables. This implies the following log-linearized relationships:

$$
\begin{aligned}
y_t^H = {} & \tilde{w}c_t + (1 - \tilde{w}) c_t^* - \left[\tilde{w}\alpha\eta + (1 - \tilde{w})(1 - \alpha^*)\eta^*\right] s_t + \\
& -\tilde{w}\kappa\delta x_t - (1 - \tilde{w})\kappa^*\delta^* x_t^*
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
y_t^{F*} = {} & \tilde{w}^* c_t + (1 - \tilde{w}^*) c_t^* + \left[\tilde{w}^*(1 - \alpha)\eta + (1 - \tilde{w}^*)\alpha^*\eta^*\right] s_t + \\
& -\tilde{w}^*\kappa\delta x_t - (1 - \tilde{w}^*)\kappa^*\delta^* x_t^*
\end{aligned}
\tag{15}
$$

whereby:

$$\tilde{w} = \frac{w(1-\alpha)(1-\kappa)}{w(1-\alpha)(1-\kappa)+(1-w)\alpha^*(1-\kappa^*)}$$

$$\tilde{w}^* = \frac{w\alpha(1-\kappa)}{w\alpha(1-\kappa)+(1-w)(1-\alpha^*)(1-\kappa^*)}$$

(16)

Market clearing conditions for the nontradable sector can be written as:

$$y_t^N = (1-\kappa)\,\delta x_t + c_t \quad y_t^{N*} = (1-\kappa^*)\,\delta^* x_t^* + c_t^* \qquad (17)$$

3.6 Monetary policy

The central bank's monetary policy is described with a Taylor (1993) rule with smoothing, which is commonly applied in the literature and empirically tested as an adequate tool for both the euro area (see e.g. Sauer and Sturm, 2003) and Poland (see i.a. Kolasa, 2009; Gradzewicz and Makarski, 2009). The common nominal interest rate is set according to the equation:

$$i_t = \rho + (1-\gamma_\rho)\,(\gamma_\pi \tilde{\pi}_t + \gamma_y \tilde{y}_t) + \gamma_\rho i_{t-1} + \varepsilon_t^i \qquad (18)$$

where i_t – central bank policy rate at t, $\tilde{y}_t$ – the output gap in a currency union, $\tilde{\pi}_t$ – inflation rate in a currency union, $\gamma_\rho \in (0;1)$ – smoothing parameter, $\gamma_\pi > 1$, $\gamma_y > 0$ – parameters of central bank's response to deviations of inflation and output from the equilibrium levels. The condition $\gamma_\pi > 1$ is necessary to satisfy the Taylor principle (Taylor, 1993), leading to a unique equilibrium.
The output gap and inflation rate for the currency union aggregate the values for individual regions, according to their size.

3.7 Labour market

We apply a simplified version of a labour market rigidity mechanism described by Erzeg, Henderson, Levin (2000) and used i.a. by Kolasa (2009). It allows the marginal rate of substitution between consumption and leisure, mrs_t, to equal the real wage, $w_t - p_t$, but only in the long run. In the short run, we let nominal wages be sticky and behave according to the Calvo scheme. Only a fraction of households, $1 - \theta^w \in (0;1)$, can renegotiate their wages in every period. This fraction remains constant and households allowed to reoptimize are selected at random. In particular, the probability of being allowed to renegotiate the wage does not depend on the amount of time elapsed since the last change. Other households partly index their their wages to past consumer inflation. Their fraction is represented by the parameter $\omega^w \in (0;1)$. Under monopolistic competition in the labour market, individual domestic and foreign households supply differentated types of labour services with the elasticity of substitution ε_w.

Solving households' optimization problem leads to the following (home) wage dynamics equation (an analogous solution holds for the foreign economy):

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \frac{(1-\theta^w)(1-\beta\theta^w)}{\theta^w[1+\phi\varepsilon_w]}[mrs_t - (w_t - p_t)] - \omega^w(\beta\pi_t - \pi_{t-1}) \quad (19)$$

3.8   Procyclical real interest rate

The mechanism of procyclical real interest rates on the country level is a specific feature of monetary unions (or, more generally, fixed exchange rate regimes). To see this, re-write the Euler equation for consumption (7) and substitute for the expected consumer price dynamics its log-linear approximations. Also, decompose expected inflation rates in individual sectors into a "forward-looking" and "backward-looking" part (a more formalized discussion of expectation types follows in Section 4):

$$c_t = \frac{h}{1+h}c_{t-1} + \frac{1}{1+h}E_t c_{t+1} + \frac{1}{\sigma}\left(\varepsilon_t^d - E_t\varepsilon_{t+1}^d\right) +$$
$$-\frac{1-h}{(1+h)\sigma}\left\{ i_t - (1-\kappa)(1-\alpha)\left[\lambda E_t\pi_{H,t+1} + (1-\lambda)\pi_{H,t-1}\right] + \right.$$
$$- (1-\kappa)\alpha\left[\lambda\pi_{F,t+1} + (1-\lambda)\pi_{F,t-1}\right] +$$
$$\left. -\kappa\left[\lambda E_t\pi_{N,t+1} + (1-\lambda)\pi_{N,t-1}\right] - \rho \right\} \quad (20)$$

The negative correlation between current inflation rate and inflation expectations becomes stronger with decreasing λ (i.e. when the backward-looking part weighs more). Also, as emphasized by European Commission (2006), the inflation expectations should be decoupled from domestic producer price dynamics when the economy is more open (i.e. high values of α and low κ in (20)). In the next Section, we investigate the extent to which the composition of expectations might affect the macroeconomic volatility.

The procyclicality of the real interest rates on the country level can also be seen as stemming from violation of the Taylor principle for an individual region (Taylor, 1993, see also an intuitive generalization by Davig and Leeper, 2007). It presumes stability of an economic system for $\gamma_\pi > 1$, i.e. central bank reacting to inflation more strongly than 1 to 1. This ensures macroeconomic stabilization through growing *ex ante* real interest rate, $i_t - E_t\pi_{t+1}$. Outside a monetary union, a national central bank that follows the Taylor rule would raise the nominal interest rate in an adequate manner. Inside a monetary union, when a rise in inflation is asymmetric, the Taylor principle may be violated. The common interest rate is set for a number of economies of different size according to their aggregate economic situation. For instance, in a 2-region monetary union, the common inflation rate is a weighted average: $\pi_t = w_1\pi_{1,t} + w_2\pi_{2,t}$. The parameter of the first country's inflation rate in the common Taylor rule equals then $\gamma_\pi w_1$ and even under $\gamma_\pi > 1$ it might not exceed unity, as long as the first economy is sufficiently small.

4 Country-level macroeconomic stability under different expectation types: simulation exercise

The model described above is used for simulating the adjustment process in the presence of asymmetric shocks. The properties of this process are characterized in this Section in two ways: by investigating (i) the impulse-response functions and (ii) second-moment properties of the variables generated with the model.

4.1 Model calibration

Table 1 describes the calibration of the model, based on three sources. Most parameters (elasticities of substitution between home and foreign tradables, Calvo probabilities, parameters of indexation, habit persistence, disutility from labour and the Taylor rule) were estimated with full information maximum likelihood method by Torój (2010). They were chosen so as to represent a "median" euro area country, i.e. as a median over the estimates in the group of 12 euro area countries. In the study of Torój (2010), the model's parameters were estimated in 12 country pairs, in which the home economy represented one of the 12 countries under consideration and the foreign economy represented the rest of the monetary union. This median country has a relative size of 3.4% of the union. Country weight, as well as α, β and κ, were calibrated in a standard way in the same article and also represent a median over the 12 countries.
Inverse intertemporal elasticity of substitution (σ) and elasticity of substitution between the tradables and nontradables (δ) were calibrated in line with Stockman and Tesar (1995). Finally, the parameters describing the stochastic properties of the disturbance vector (serial correlations and variances) are based on the values obtained by Kolasa (2009) for the euro area. Wherever possible, the home and foreign economy are described by the same parameter values.

4.2 Simulation scenarios: types of expectations

In the simulations, we apply 5 basic types of expectations commonly considered in the literature. Following Pilbeam (2006), we specify:

° static expectations:

$$E_t v_{t+1} = v_t \tag{21}$$

Economic agents expect that variable v shall remain at the current level at $t + 1$. As macroeconomic data is normally released with a lag, the specification $E_t v_{t+1} = v_{t-1}$ seems to be more realistic. A one-period lag should also remain generally compatible with a quarterly frequency of the model's variables (production, consumption, inflation rate).

Table 1: Calibration of the model

parameter	value	parameter	value	parameter	value
η	1.666	$\rho_d = \rho_d^*$	0.65	$\phi = \phi^*$	0.841
η^*	1.275	$\rho_H = \rho_F$	0.62	γ_ρ	0.704
$\omega_H = \omega_F$	0.391	$\rho_N = \rho_N^*$	0.70	γ_π	1.795
$\omega_N = \omega_N^*$	0.147	$\rho_W = \rho_W^*$	0.62	γ_y	0.482
$\omega_W = \omega_W^*$	0.098	$\rho_i = \rho_i^*$	0	$\beta = \beta^*$	0.995
$\theta_H = \theta_F$	0.447	$\sigma_d^2 = \sigma_d^{2*}$	2.34^2	α	0.443
$\theta_N = \theta_N^*$	0.698	$\sigma_H^2 = \sigma_F^2$	2.08^2	α^*	0.047
$\theta_W = \theta_W^*$	0.518	$\sigma_N^2 = \sigma_N^{2*}$	0.91^2	$\kappa = \kappa^*$	0.765
$h = h^*$	0.770	$\sigma_W^2 = \sigma_W^{2*}$	5.5^2	w	0.034
$\delta = \delta^*$	0.5	$\sigma_i^2 = \sigma_i^{2*}$	0.09^2	ε^W	3.000
$\sigma = \sigma^*$	2.0				

$\circ$ adaptive expectations:

$$E_t v_{t+1} = a \cdot v_t + (1 - a) \cdot E_{t-1} v_t \qquad (22)$$

whereby $0 < a < 1$. This type of expectations represents an inertial process, in which agents update their beliefs in every period. It is characterized by the coefficient a that weighs information acquired at t (or $t - 1$, when the data reading is lagged). Iterating equation (22) backwards, we conclude that adaptive expectations – unlike static ones – apply non-zero weights not only to values at $t - 1$, but also in preceding periods.

$\circ$ extrapolative expectations:

$$E_t v_{t+1} = v_t + a (v_t - v_{t-1}) \qquad (23)$$

whereby $a > 0$. Economic agents specify the expected value of v one period ahead, extrapolating its current value using part of its recent dynamics.

$\circ$ regressive expectations:

$$E_t v_{t+1} = a v_t + (1 - a) \bar{v} \qquad (24)$$

whereby $0 < a < 1$ and $\bar{v}$ denotes the steady-state value of v. Specification (24) captures economic agents' perception of a variable as mean-reverting with some degree of inertia. Like in case of static expectations, we replace v_t by v_{t-1} due to lags in macroeconomic readings and finally obtain $E_t v_{t+1} = a v_{t-1} + (1 - a) \bar{v}$.

$\circ$ rational expectations:

$$E_t^R v_{t+1} = v_{t+1} + u_{t+1} \qquad (25)$$

whereby $E(u_{t+1}) = 0$. This means that agents do not commit systematic errors in the process of forming expectations.

In the case of parametrized expectation types, we consider $a = 0.33$ or $a = 0.66$ for extrapolative and regressive expectations, as well as $a = 0.5$ for adaptive ones. In the two-variant cases, this allows us to take into consideration the strength of anchoring around the steady state level and of extrapolation (respectively) as additional determinants of the adjustment process in the analysis. This constitutes 7 scenarios with homogenous expectations.

On top of that, we take into account 6 mixed-type expectations, in which rational expectations enter with a weight 0.5 and the other half is represented by any of the other types:

$$E_t v_{t+1} = 0.5 \cdot E_t^R v_{t+1} + 0.5 \cdot E_t^m v_{t+1}$$

with index m representing extrapolative, regressive, adaptive or static expectations (with a consistent parametrization). This can be viewed as either an *ad hoc* representation of all agents' expectations, being represented by a convex linear combination of basic theoretical mechanisms, or as a representation of heterogenous agents in the economy. However, the latter interpretation requires that necessary aggregation conditions hold (see 7 conditions enumerated by Branch and McGough, 2009, for their technical exposition). The aggregation of heterogenous expectations requires i.a. that the average of agents' self-expectations equals the joint average of self- and cross-expectations (see Koloch, 2010, for more discussion). Finally, with the 7 homogenous specifications, this yields 13 types for simulation analysis.

The impulse-response functions, presented in Figures 1-4, were calculated under the assumption that all agents within the economy represent the same type of expectations. In further tables, we consider all 13^2 combinations resulting from differentiation of expectation types between two groups: (i) domestic and foreign agents and (ii) producers and consumers. The above differentiation was implemented by substituting appropriate specifications into equations that contain expectational terms by the type of respective decision-taking agents. Specifically, domestic agents' expectations are reflected in country-specific Euler equations, Phillips curves and wage equation. On the other hand, producers' expectations are included in the pricesetting mechanism that underlies the Phillips curves, whereby it is households' decision problem that leads to the Euler equation for consumption, as well as wage equation.

4.3 Model stability

Simulation exercises can only be completed when the model structure guarantees an appropriate solution. In the scenarios with at least partly rational expectations, this requires that Blanchard-Kahn (1980) conditions be satisfied so that there exists a unique saddle-path stable solution. Wherever any form of rational expectations is absent from the model, its log-linear approximation can be directly represented as a SVAR(1) process, premultiplied by the inverse of the matrix of parameters for the expectational terms. In this case, however, the stability of the system needs to

be investigated, i.e. whether the companion matrix of the system does or does not contain explosive eigenvalues.

It turns out that for some types of expectations (or their combinations), the system does not fulfil its appropriate stability condition (see Table 2):

1. When all agents (domestic and foreign, consumers and producers) represent the same type of expectations, the model is unstable for adaptive, static or extrapolative expectations. Note that this is not the case when these types are combined with rational expectations in equal proportions.

2. When we differentiate between domestic and foreign agents, the type of expectations prevailing in the home economy (intuitively) determines whether the model is stable or not. Namely, Blanchard-Kahn conditions are violated because of an excessive number of unstable roots when domestic agents' expectations are adaptive, extrapolative or static. This can be explained by the fact that adjustments in the foreign economy are to a large extent supported by the common monetary policy, which is not the case for the relatively small, home economy. However, the model's instability can also originate in the foreign economy, as long as it represents agents with extrapolative expectations.

3. A more nuanced picture emerges from the differentiation between consumers and producers (see Table 2). Extrapolative expectations still induce the model's instability, regardless of the value of a and the group of agents that forms them. Moreover, a high number of unstable models involves either static or adaptive expectations in one of the groups. Note, however, that under rational expectations in one of the groups, the model always fulfils the stability conditions (except for the previously mentioned case of extrapolative expectations in the other group). It should perhaps be stressed that partly rational expectations also highly improve the model's stability.

The results from Table 2 cannot be regarded as completely discarding the Walters critique. In fact, the model can be seen as inherently unstable for some of the combinations. This does not necessarily result from the participation in a monetary union. However, the differentiation between domestic and foreign agents reveals that an autonomous monetary policy (or the common monetary policy from the perspective of a big economy in a monetary union) can be seen as a strongly stabilizing mechanism when the type of expectations does not support the adjustment process. In particular, adaptive or static expectations turned out to induce instability in the small home economy, but ensure stability in the big foreign economy.

4.4   Impulse-response analysis

Figures 1-4 present the impulse-response functions after 4 types of asymmetric shocks in the economy: demand, supply (in tradable and nontradable sector) and labour supply. In this part of the analysis, homogenous types of expectations are assumed (both

Table 2: Existence of stable solutions under different expectations

(a) home vs foreign agents' expectations

	foreign country												
home country	A	B	C1	C2	D1	D2	E	F	G	H	I	J	K
(A) rational	1	1	0	0	1	1	1	1	1	1	1	1	1
(B) adaptive ($a = 0.5$)	0	0	0	0	0	0	0	0	0	0	0	0	0
(C1) extrapolative ($a = 0.66$)	0	0	0	0	0	0	0	0	0	0	0	0	0
(C2) extrapolative ($a = 0.33$)	0	0	0	0	0	0	0	0	0	0	0	0	0
(D1) regressive ($a = 0.66$)	1	1	0	0	1	1	1	1	1	1	1	1	1
(D2) regressive ($a = 0.33$)	1	1	0	0	1	1	1	1	1	1	1	1	1
(E) static	0	0	0	0	0	0	0	0	0	0	0	0	0
(F) $0.5A + 0.5B$	1	1	0	0	1	1	1	1	1	1	1	1	1
(G) $0.5A + 0.5C1$	1	1	0	0	1	1	1	1	1	1	1	1	1
(H) $0.5A + 0.5C2$	1	1	0	0	1	1	1	1	1	1	1	1	1
(I) $0.5A + 0.5D1$	1	1	0	0	1	1	1	1	1	1	1	1	1
(J) $0.5A + 0.5D2$	1	1	0	0	1	1	1	1	1	1	1	1	1
(K) $0.5A + 0.5E$	1	1	0	0	1	1	1	1	1	1	1	1	1

(b) producers' vs consumers' expectations

	consumers												
producers	A	B	C1	C2	D1	D2	E	F	G	H	I	J	K
(A) rational	1	1	0	0	1	1	1	1	1	1	1	1	1
(B) adaptive ($a = 0.5$)	1	0	0	0	1	1	0	1	1	1	1	1	0
(C1) extrapolative ($a = 0.66$)	0	0	0	0	0	0	0	0	0	0	0	0	0
(C2) extrapolative ($a = 0.33$)	0	0	0	0	0	0	0	0	0	0	0	0	0
(D1) regressive ($a = 0.66$)	1	1	0	0	1	1	1	1	1	1	1	1	1
(D2) regressive ($a = 0.33$)	1	1	0	0	1	1	1	1	1	1	1	1	1
(E) static	1	0	0	0	1	0	0	0	0	0	0	0	0
(F) $0.5A + 0.5B$	1	1	0	0	1	1	1	1	1	1	1	1	1
(G) $0.5A + 0.5C1$	1	0	0	0	0	0	0	1	1	1	0	1	1
(H) $0.5A + 0.5C2$	1	0	0	0	0	0	0	1	1	1	0	0	1
(I) $0.5A + 0.5D1$	1	1	0	0	1	1	1	1	1	1	1	1	1
(J) $0.5A + 0.5D2$	1	1	0	0	1	1	1	1	1	1	1	1	1
(K) $0.5A + 0.5E$	1	1	0	0	1	1	1	1	1	1	1	1	1

1 – model can be solved/stable; 0 – model does not fulfil Blanchard-Kahn conditions/unstable.

for producers and consumers, as well as home and domestic agents). Scenarios B, C and E are not included in the Figures because of the model's instability. Scenarios G and H were not presented because these types imply an incomparably higher volatility.

For every type of asymmetric shock under consideration, the adjustment paths differ considerably (in both quantitative and, more often than not, qualitative terms) with

the type of expectations applied in the model.

In the aftermath of an asymmetric demand shock hitting the home economy, only the level of consumption (and hence nontradable production) remains insensitive to the choice of the scenario. All other depicted variables (tradable production, inflation of tradables and nontradables, internal and external terms of trade, real wages) adjust most efficiently to the equilibrium level in the scenario where all agents are rational (A). The other extreme is represented by the scenario in which expectations are half-rational and half-static (K). It can be characterized by both the highest amplitude and, in some cases, a significant lag. A comparable picture emerges from the analy-

Figure 1: Response to an asymmetric demand shock
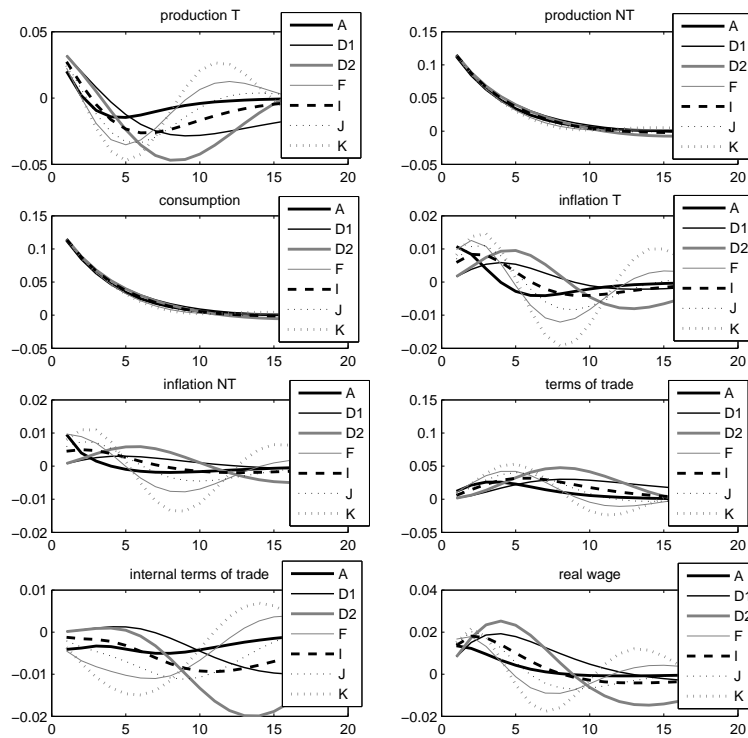
sis of impulse-response functions after an asymmetric supply shocks in both sectors. For most of the variables, rational expectations (A) ensure the lowest macroeconomic volatility and the most efficient adjustment process. However, a remarkable pattern of consumption is noteworthy. A positive (but short-lived) supply shock generates

a short-lived drop in inflation rate, associated with higher inflation expectations for the future. They lower the *ex ante* real interest rate and support a boom in consumption. In this case, rational expectations fail to contain the boom, but succeed in ensuring a relatively quick realignment to the equilibrium level. On the other hand, the boom is to a large extent contained by the regressive expectations, especially when they are parametrized so as to ensure a strong anchoring around an equilibrium level ($a = 0.33$, D1). If an asymmetric supply shock originates in the nontradable sector, the adjustment of the consumption under rational expectations resembles the one in D1 scenario and clearly outpaces other scenarios.

Figure 2: Response to an asymmetric supply shock in the tradable sector
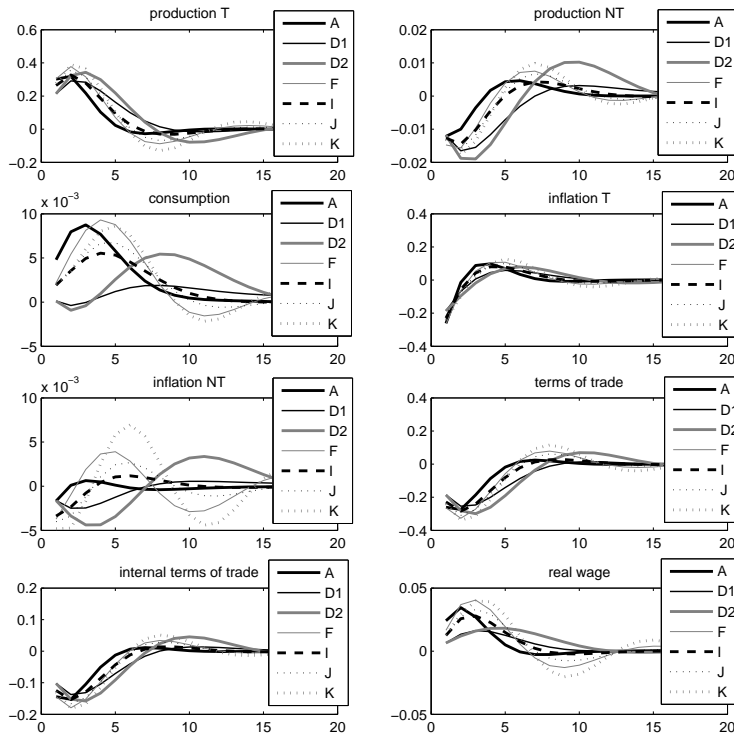
The economy's response to an asymmetric adverse labour supply shock (Figure 4) also seems to generate the least macroeconomic volatility under rational expectations. Rational agents anticipate the loss in competitiveness associated with a sudden increase in wages and prices and, consequently, the wage pressure quickly subsides. The

Figure 3: Response to an asymmetric supply shock in the nontradable sector
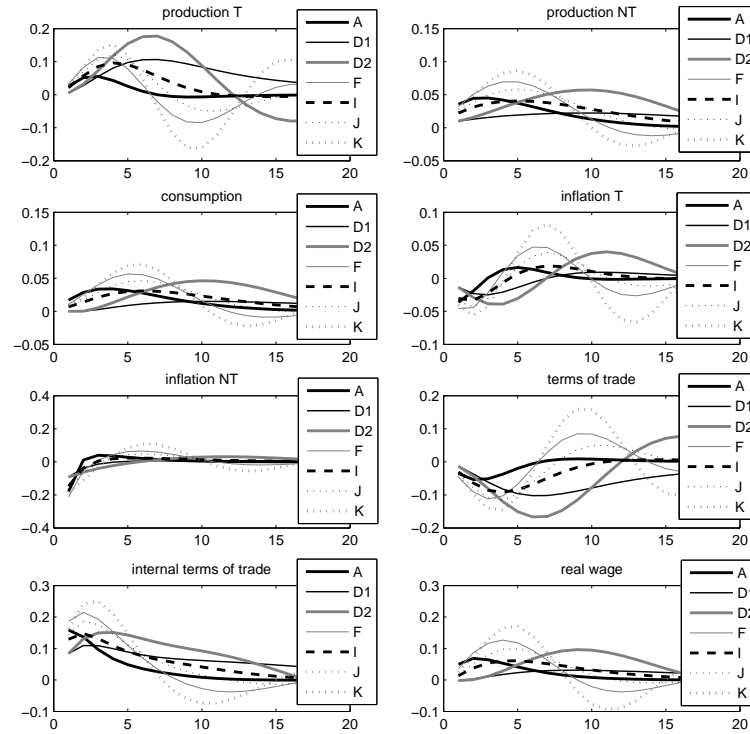
households' wage requirements fall, which outpaces the competitive adjustment. Consequently, the economy avoids a long recession, associated with real depreciation. The adjustment runs less efficiently if the link between current levels of macroeconomic variables and their expected values is stronger. The procyclicality of *ex ante* real interest rate is directly illustrated in Figure 5. A positive demand shock in the home economy markedly lowers this rate, unless expectations are strongly anchored (D1). The reversal is relatively quick under rational expectations. Weakly anchored, regressive expectations (D2) imply a slower reduction in country real interest rate, followed by a gradual increase. The other scenarios imply lasting boom-bust episodes, whereby their highest amplitude (in terms of real interest rate volatility) can be attributed to the scenario of partly static expectations. A similar pattern can be observed after the other 3 asymmetric shocks under consideration. Interesting qualitative differences, however, can be observed for asymmetric supply shocks in both sectors. A positive supply shock reduces the inflation rate. This implies a (sometimes sharp) rise in

Figure 4: Response to an asymmetric labour supply shock
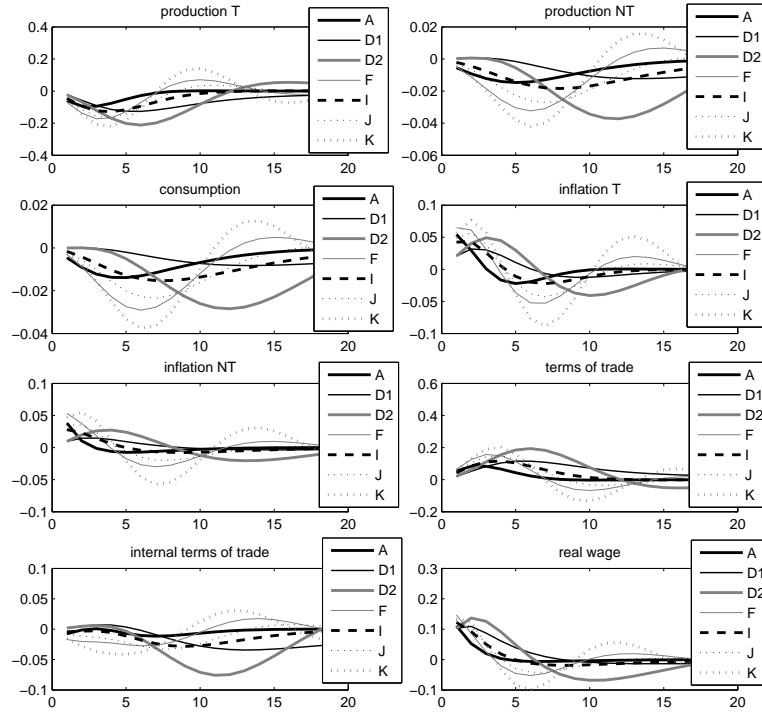
real interest rates, as long as backward-looking types of expectations are applied. Under rational expectations, agents anticipate a subsequent rise in inflation as the shock gradually dies out and the *ex ante* real interest rate falls already on impact. Remarkably, rational expectations do not imply an exceptionally low correlation (in terms of magnitude) between the *ex ante* home real interest rate and home consumer price dynamics (see Table 3). A significantly negative correlation coefficient (-0.3 with a standard deviation of order 0.0001 across simulations) implies that rationality does not eliminate the problem of real interest rate procyclicality. Two points can be made here, however. Firstly, it substantially alleviates this problem as compared to the case of partly extrapolative expectations. Secondly, the efficiency of rational expectations as a tool enhancing the country-specific adjustment capacity does only result from reducing the real interest rate fluctuations of procyclical character, but also – to a substantial extent – from increasing the efficiency of the competitiveness channel.

Figure 5: *Ex ante* real interest rates after asymmetric shocks for different expectation types
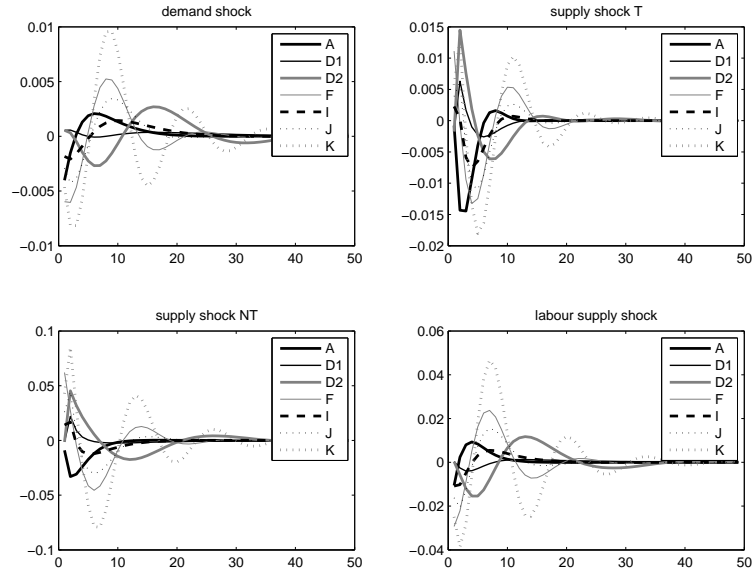
4.5 Macroeconomic volatility under different expectation types

The insights from the IRF analysis can be completed by comparing the volatility of macroeconomic variables in the model across 13 types of expectations under consideration. For this purpose, 50 paths of 10000 observations of the (normally distributed) structural disturbance vector were generated. The model was simulated using these paths under different types of expectations (every time, the same path was used for all the scenarios under consideration). The volatilities of macroeconomic variables were calculated and then averaged over the 50 paths.

Table 4 presents the variance of individual variables under 13 scenarios in question with full homogeneity of expectation types across groups. In accordance with the previous conclusions from the IRF analysis, rational expectations yield the lowest variance for most of the variables. Regressive expectations (but only with lower a, i.e. stronger anchoring around the steady-state levels) rank best in terms of inflation rate volatility. However, this improvement results in an increasing macroeconomic volatility in the other categories.

At the other extreme, (partly) extrapolative expectations rank worst in terms of macroeconomic volatility, followed by (partly) static ones. Intiutively, the higher value of parameter a for extrapolative expectations, the higher was the variance of

Table 3: Correlation between *ex ante* real interest rate and consumer inflation rate under different expectations

(a) home vs foreign agents' expectations

home country	foreign country										
	A	B	D1	D2	E	F	G	H	I	J	K
(A) rational	-0.30	0.69	-0.35	-0.01	0.84	0.45	-0.09	-0.24	-0.13	0.13	0.47
(B) adaptive ($a = 0.5$)											
(C1) extrapolative ($a = 0.66$)											
(C2) extrapolative ($a = 0.33$)											
(D1) regressive ($a = 0.66$)	-0.01	0.22	0.20	0.33	-0.06	-0.16	-0.27	-0.16	0.24	0.21	-0.37
(D2) regressive ($a = 0.33$)	-0.63	0.50	-0.59	-0.32	0.21	-0.36	-0.64	-0.65	-0.45	-0.32	-0.55
(E) static											
(F) $0.5A + 0.5B$	-0.76	0.66	-0.79	-0.46	0.84	0.27	-0.59	-0.70	-0.65	-0.33	0.69
(G) $0.5A + 0.5C1$	-0.97	-0.56	-0.95	-0.92	-0.34	-0.84	-0.97	-0.97	-0.94	-0.91	-0.63
(H) $0.5A + 0.5C2$	-0.98	-0.80	-0.96	-0.95	-0.73	-0.92	-0.98	-0.98	-0.95	-0.94	-0.81
(I) $0.5A + 0.5D1$	-0.26	0.72	-0.15	0.29	0.75	0.37	-0.25	-0.30	0.06	0.28	0.45
(J) $0.5A + 0.5D2$	-0.61	0.76	-0.62	-0.15	0.85	0.43	-0.46	-0.57	-0.41	-0.03	0.68
(K) $0.5A + 0.5E$	-0.92	0.03	-0.93	-0.83	0.29	-0.45	-0.85	-0.89	-0.89	-0.78	-0.06

(b) producers' vs consumers' expectations

producers	consumers										
	A	B	D1	D2	E	F	G	H	I	J	K
(A) rational	-0.30	-0.63	-0.48	-0.53	-0.73	-0.44	-0.31	-0.22	-0.45	-0.46	-0.47
(B) adaptive ($a = 0.5$)	0.31		-0.43	-0.66		-0.62	0.23	0.32	-0.40	-0.56	
(D1) regressive ($a = 0.66$)	0.30	0.08	0.20	0.16	0.01	0.17	0.25	0.28	0.21	0.18	0.11
(D2) regressive ($a = 0.33$)	0.15	-0.58	-0.21	-0.32	-0.80	-0.18	0.03	0.11	-0.11	-0.17	-0.30
(E) static	-0.81		-0.90								
(F) $0.5A + 0.5B$	0.18	0.07	-0.05	-0.02	-0.34	0.27	0.16	0.17	0.06	0.13	0.24
(G) $0.5A + 0.5C1$	-0.98					-0.97	-0.97	-0.98		-0.98	-0.96
(H) $0.5A + 0.5C2$	-0.98					-0.97	-0.97	-0.98			-0.95
(I) $0.5A + 0.5D1$	0.24	-0.02	0.00	0.00	-0.20	0.16	0.20	0.25	0.06	0.08	0.10
(J) $0.5A + 0.5D2$	0.16	-0.22	-0.14	-0.16	-0.50	0.07	0.13	0.18	-0.05	-0.03	-0.01
(K) $0.5A + 0.5E$	0.09	-0.17	-0.10	-0.23	-0.44	0.04	0.05	0.08	-0.02	-0.04	-0.06

the other variables. Furthermore, adaptive expectations (scenario B) rank better or worse that regressive expectations (scenario J), according to which variable is taken into consideration. In general, they shift part of the volatility from quantities to prices, lowering the variance of nontradable output and consumption, but boosting the variance of inflation rates, price relations and hence the tradable output.

Differentiating between the type of expectations among domestic and foreign agents leads to a conclusion that, in general, the combination of rational agents both at home and abroad yields the lowest macroeconomic volatility (see Tables 5-8). This holds without any exception for consumption, internal terms of trade and real wages. Few combinations rank better for individual variables, including tradable output (domestic: A, foreign: D1), nontradable output (domestic: A, foreign G) and external terms of trade (domestic: A, foreign: D1). Note that these combinations include either A

Table 4: Variance of individual variables under different expectation types (rescaled: A=1)

Expectations	y^T	y^{NT}	c	π^T	π^{NT}	s	x	rw
(A) rational	**1.00**	**1.00**	**1.00**	1.00	1.00	**1.00**	**1.00**	**1.00**
(B) adaptive ($a = 0.5$)	-	-	-	-	-	-	-	-
(C1) extrapolative ($a = 0.66$)	-	-	-	-	-	-	-	-
(C2) extrapolative ($a = 0.33$)	-	-	-	-	-	-	-	-
(D1) regressive ($a = 0.33$)	2.83	2.65	2.02	**0.75**	**0.60**	2.59	3.43	3.91
(D2) regressive ($a = 0.66$)	7.13	7.28	5.23	2.91	4.92	6.35	8.11	13.04
(E) static	-	-	-	-	-	-	-	-
(F) $0.5A + 0.5B$	3.22	2.13	1.70	3.51	7.27	2.92	3.33	3.29
(G) $0.5A + 0.5C1$	94.98	24.25	40.64	19.39	59.09	87.93	54.46	20.38
(H) $0.5A + 0.5C2$	587.91	80.43	171.26	38.01	145.45	546.40	421.79	74.06
(I) $0.5A + 0.5D1$	1.96	2.13	1.72	1.13	1.19	1.82	2.40	2.55
(J) $0.5A + 0.5D2$	2.79	2.37	1.87	2.22	3.35	2.52	2.92	3.38
(K) $0.5A + 0.5E$	8.54	6.13	4.51	17.70	47.51	6.86	6.52	15.93

Identical process of expectation formation assumed between home and foreign agents, as well as producers and consumers.

(i.e. rational expectations – at home) or D1 (i.e. strongly anchored expectations – abroad).

In case of tradable and nontradable inflation rates, there are more combinations ranking better in terms of low variance than A at home and A abroad. They include:

° for π^T: **A**(domestic) + G or H (foreign); **D1**(domestic) + A, D1, D2, I or J (foreign);

° for π^{NT}: **A**(domestic) + D1, G or H (foreign); **D1**(domestic) + A, D1, D2, I or J (foreign).

This result means that inflation volatility at home could be reduced under regressive expectations (as compared to rational) provided that:

1. expectations abroad are either rational or regressive (or a combination of both types);

2. a cost of rising volatility for other variables is taken into account.

Intuitively, the type of domestic agents' expectations seems to influence the domestic macroeconomic volatility to a larger extent than foreign agents'. However, it shoud be stressed that the impact of the latter type can be seen as non-negligible.

Table 5: Variance of tradable and nontradable output under different expectation types of home vs foreign agents (rescaled: A vs A =1)

y^T

home country		foreign country												
	A	B	C1	C2	D1	D2	E	F	G	H	I	J	K
(A) rational	**1.00**	2.46			**0.95**	1.21	4.69	2.32	1.34	1.19	1.02	1.31	6.58
(B) adaptive ($a = 0.5$)													
(C1) extrapolative ($a = 0.66$)													
(C2) extrapolative ($a = 0.33$)													
(D1) regressive ($a = 0.33$)	3.37	70.58			2.83	4.12	41.25	14.46	5.56	4.39	3.35	4.96	29.80
(D2) regressive ($a = 0.66$)	5.38	25.49			5.46	7.13	27.40	18.45	7.45	6.23	5.78	7.74	49.38
(E) static													
(F) $0.5A + 0.5B$	2.32	3.20			2.25	2.45	4.12	3.22	2.67	2.51	2.33	2.60	5.86
(G) $0.5A + 0.5C1$	97.03	122.94			55.49	56.73	199.04	92.55	94.98	112.04	60.33	65.65	107.38
(H) $0.5A + 0.5C2$	552.61	650.99			301.00	310.45	790.87	520.37	503.04	587.91	331.67	367.89	540.27
(I) $0.5A + 0.5D1$	1.88	16.12			1.74	2.41	41.05	6.02	2.59	2.23	1.96	2.70	15.21
(J) $0.5A + 0.5D2$	2.25	6.23			2.11	2.45	12.85	4.49	2.80	2.51	2.29	2.79	10.84
(K) $0.5A + 0.5E$	4.07	6.09			4.12	4.57	8.43	5.82	4.76	4.30	4.36	4.95	8.54

y^{NT}

home country		foreign country												
	A	B	C1	C2	D1	D2	E	F	G	H	I	J	K
(A) rational	**1.00**	1.73			1.11	1.10	3.00	1.27	**0.96**	1.01	1.12	1.14	1.81
(B) adaptive ($a = 0.5$)													
(C1) extrapolative ($a = 0.66$)													
(C2) extrapolative ($a = 0.33$)													
(D1) regressive ($a = 0.66$)	5.00	51.69			2.65	4.16	40.08	9.92	6.61	6.74	3.18	4.77	15.91
(D2) regressive ($a = 0.33$)	5.17	32.95			5.19	7.28	30.70	14.86	6.45	5.98	5.49	7.27	29.07
(E) static													
(F) $0.5A + 0.5B$	1.71	1.98			1.78	1.81	2.08	2.13	1.67	1.68	1.81	1.88	3.29
(G) $0.5A + 0.5C1$	26.13	28.04			19.77	19.50	39.23	24.37	24.25	28.64	20.70	20.98	26.41
(H) $0.5A + 0.5C2$	76.27	87.62			55.13	54.77	106.40	73.82	69.31	80.43	58.31	60.44	76.68
(I) $0.5A + 0.5D1$	2.28	20.42			1.86	2.48	49.83	3.74	2.24	2.50	2.13	2.70	4.91
(J) $0.5A + 0.5D2$	1.96	7.33			1.95	2.31	15.46	3.02	1.91	1.99	2.08	2.37	4.77
(K) $0.5A + 0.5E$	2.41	3.45			2.48	2.75	4.29	3.46	2.54	2.44	2.60	2.89	6.13

A. Torój

Table 6: Variance of consumption and real wages under different expectation types of home vs foreign agents (rescaled: A vs A =1)

c

home country	A	B	C1	C2	D1	D2	E	F	G	H	I	J	K
							foreign country						
(A) rational	**1.00**	1.41			1.07	1.06	2.14	1.07	1.01	1.04	1.06	1.05	1.27
(B) adaptive ($a = 0.5$)													
(C1) extrapolative ($a = 0.66$)													
(C2) extrapolative ($a = 0.33$)													
(D1) regressive ($a = 0.33$)	4.23	30.86			2.02	2.94	28.39	7.90	5.90	5.93	2.37	3.45	14.16
(D2) regressive ($a = 0.66$)	3.99	19.85			3.84	5.23	19.84	10.35	5.08	4.73	4.03	5.23	20.67
(E) static													
(F) $0.5A + 0.5B$	1.54	1.72			1.61	1.61	1.87	1.70	1.50	1.52	1.61	1.62	2.14
(G) $0.5A + 0.5C1$	43.13	48.39			30.25	30.09	70.77	40.42	40.64	47.88	31.97	33.03	44.31
(H) $0.5A + 0.5C2$	162.08	187.23			105.45	106.11	226.68	155.03	147.27	171.26	113.17	120.24	160.28
(I) $0.5A + 0.5D1$	1.97	12.24			1.55	1.88	28.85	2.76	2.03	2.23	1.72	2.07	3.68
(J) $0.5A + 0.5D2$	1.69	4.43			1.66	1.84	8.61	2.15	1.67	1.75	1.73	1.87	2.86
(K) $0.5A + 0.5E$	2.07	2.95			2.13	2.31	3.76	2.77	2.17	2.10	2.21	2.39	4.51

rw

home country	A	B	C1	C2	D1	D2	E	F	G	H	I	J	K
							foreign country						
(A) rational	**1.00**	1.37			1.03	1.04	1.98	1.06	1.02	1.03	1.03	1.04	1.25
(B) adaptive ($a = 0.5$)													
(C1) extrapolative ($a = 0.66$)													
(C2) extrapolative ($a = 0.33$)													
(D1) regressive ($a = 0.66$)	5.97	43.81			3.91	5.10	36.96	11.46	7.73	7.58	4.35	5.76	20.80
(D2) regressive ($a = 0.33$)	10.33	35.10			10.00	13.04	40.85	25.08	12.98	11.80	10.49	13.23	55.77
(E) static													
(F) $0.5A + 0.5B$	2.81	3.15			2.90	2.92	3.39	3.29	2.76	2.77	2.92	2.99	4.71
(G) $0.5A + 0.5C1$	21.97	23.28			15.90	15.70	32.79	20.33	20.38	24.40	16.78	17.10	22.77
(H) $0.5A + 0.5C2$	70.13	80.01			48.67	48.48	97.15	67.54	63.52	74.06	51.82	54.14	70.58
(I) $0.5A + 0.5D1$	2.74	15.96			2.35	2.77	37.67	4.01	2.83	2.97	2.55	2.99	5.55
(J) $0.5A + 0.5D2$	3.07	6.94			3.02	3.29	12.92	4.01	3.10	3.13	3.14	3.38	5.70
(K) $0.5A + 0.5E$	6.25	8.89			6.47	7.18	10.99	9.12	6.75	6.34	6.79	7.61	15.93

Table 7: Variance of tradable and nontradable inflation under different expectation types of home vs foreign agents (rescaled: A vs A =1)

π^T

home country	A	B	C1	C2	D1	D2	E	F	G	H	I	J	K
							foreign country						
(A) rational	**1.00**	4.00			1.01	1.14	8.31	1.34	**0.95**	**0.99**	1.07	1.20	1.66
(B) adaptive ($a = 0.5$)													
(C1) extrapolative ($a = 0.66$)													
(C2) extrapolative ($a = 0.33$)													
(D1) regressive ($a = 0.33$)	**0.91**	3.50			**0.75**	**0.85**	3.11	1.38	1.05	1.05	**0.79**	**0.91**	2.37
(D2) regressive ($a = 0.66$)	2.36	7.38			2.30	2.91	8.56	5.11	2.84	2.62	2.40	2.94	11.34
(E) static													
(F) $0.5A + 0.5B$	2.30	5.84			2.33	2.52	10.80	3.51	2.27	2.26	2.43	2.68	6.72
(G) $0.5A + 0.5C1$	21.96	25.72			16.51	16.35	37.05	20.20	19.39	23.83	17.64	17.97	21.31
(H) $0.5A + 0.5C2$	36.60	44.42			32.09	31.48	52.57	36.40	32.29	38.01	33.80	33.97	37.00
(I) $0.5A + 0.5D1$	1.15	4.52			1.07	1.20	10.02	1.66	1.17	1.20	1.13	1.26	2.35
(J) $0.5A + 0.5D2$	1.87	5.49			1.87	2.15	10.96	2.94	1.90	1.89	1.97	2.22	5.08
(K) $0.5A + 0.5E$	4.79	10.56			5.13	6.16	17.38	8.54	5.21	4.79	5.47	6.49	17.70

π^{NT}

home country	A	B	C1	C2	D1	D2	E	F	G	H	I	J	K
							foreign country						
(A) rational	**1.00**	13.32			**0.97**	1.55	30.63	1.96	**0.85**	**0.98**	1.23	1.72	2.47
(B) adaptive ($a = 0.5$)													
(C1) extrapolative ($a = 0.66$)													
(C2) extrapolative ($a = 0.33$)													
(D1) regressive ($a = 0.66$)	**0.85**	10.08			**0.60**	**0.86**	6.91	1.80	1.05	1.04	**0.69**	**0.95**	2.77
(D2) regressive ($a = 0.33$)	3.45	22.98			3.62	4.92	20.92	10.36	4.17	3.79	3.81	4.97	20.45
(E) static													
(F) $0.5A + 0.5B$	3.43	19.00			3.48	4.26	41.23	7.27	3.29	3.33	3.84	4.70	16.07
(G) $0.5A + 0.5C1$	67.54	87.56			47.31	47.46	134.54	62.22	59.09	73.64	51.55	53.76	65.31
(H) $0.5A + 0.5C2$	140.12	172.83			117.48	115.99	205.26	138.29	122.68	145.45	125.25	127.56	139.94
(I) $0.5A + 0.5D1$	1.19	14.24			1.03	1.46	35.79	2.42	1.19	1.26	1.19	1.57	3.49
(J) $0.5A + 0.5D2$	2.39	16.73			2.43	3.23	39.29	5.10	2.37	2.40	2.68	3.35	9.42
(K) $0.5A + 0.5E$	9.22	30.33			10.33	13.47	56.65	20.06	9.84	9.06	11.20	13.96	47.51

Table 8: Variance of external and internal terms of trade under different expectation types of home vs foreign agents (rescaled: A vs A =1)

s

home country		foreign country											
	A	B	C1	C2	D1	D2	E	F	G	H	I	J	K
(A) rational	1.00	2.84			0.95	1.36	5.37	2.51	1.42	1.25	1.03	1.41	6.77
(B) adaptive ($a = 0.5$)													
(C1) extrapolative ($a = 0.66$)													
(C2) extrapolative ($a = 0.33$)													
(D1) regressive ($a = 0.33$)	2.98	62.30			2.59	3.99	37.31	13.22	4.97	3.93	3.09	4.70	27.50
(D2) regressive ($a = 0.66$)	4.47	22.09			4.60	6.35	24.70	16.02	6.25	5.20	4.95	6.81	42.76
(E) static													
(F) $0.5A + 0.5B$	2.00	3.27			1.94	2.27	4.48	2.92	2.39	2.24	2.01	2.33	5.18
(G) $0.5A + 0.5C1$	89.87	113.40			52.64	53.78	182.23	86.08	87.93	103.47	56.95	61.72	100.05
(H) $0.5A + 0.5C2$	513.85	604.96			283.53	291.96	734.68	484.89	467.92	546.40	311.66	344.70	503.71
(I) $0.5A + 0.5D1$	1.68	14.28			1.59	2.42	35.17	5.62	2.36	2.03	1.82	2.63	14.11
(J) $0.5A + 0.5D2$	1.92	5.67			1.77	2.24	11.18	4.03	2.44	2.19	1.97	2.52	9.59
(K) $0.5A + 0.5E$	3.33	5.20			3.28	3.66	7.29	4.80	3.98	3.59	3.51	4.05	6.86

x

home country		foreign country											
	A	B	C1	C2	D1	D2	E	F	G	H	I	J	K
(A) rational	1.00	4.87			1.05	1.61	10.56	3.60	1.31	1.16	1.19	1.76	9.81
(B) adaptive ($a = 0.5$)													
(C1) extrapolative ($a = 0.66$)													
(C2) extrapolative ($a = 0.33$)													
(D1) regressive ($a = 0.66$)	3.30	122.20			3.43	6.05	57.32	14.07	4.46	3.84	4.33	6.77	20.24
(D2) regressive ($a = 0.33$)	4.85	58.30			5.61	8.11	46.88	20.91	6.01	5.24	6.10	8.55	39.68
(E) static													
(F) $0.5A + 0.5B$	1.63	3.41			1.58	2.07	4.78	3.33	1.95	1.81	1.73	2.21	8.26
(G) $0.5A + 0.5C1$	54.40	76.23			30.02	31.52	125.11	54.38	54.46	63.18	32.79	36.72	63.48
(H) $0.5A + 0.5C2$	396.18	473.87			203.35	212.52	571.23	373.65	361.54	421.79	226.00	255.36	387.39
(I) $0.5A + 0.5D1$	1.81	39.20			1.98	3.62	101.40	7.58	2.19	1.99	2.40	3.72	15.83
(J) $0.5A + 0.5D2$	1.72	14.11			1.73	2.69	32.81	5.60	2.06	1.89	2.02	2.92	14.25
(K) $0.5A + 0.5E$	2.12	3.79			2.02	2.36	5.82	3.44	2.47	2.29	2.21	2.65	6.52

The rationality of all agents is also the scenario ensuring lowest macroeconomic volatility when we consider a differentiation between the type of expectations represented by producers and consumers in both regions. Combination of rational consumers and rational producers resulted in lowest varaince of tradable and nontradable output, consumption, as well as external and internal terms of trade (see Tables 9-12). However, in this case a few exceptions could be noted if the volatility of inflation rates and real wages alone were used as a criterion. The best scenario supporting low inflation volatility turned out to be a combination of rational producers (A) and strongly regressive consumers (D1). This result could be explained by specific features of the monetary union. On the one hand, rational producers are more aware of future demand constraints associated with a possible loss (or gain) in external competitiveness and hence anticipate the functioning of the competitiveness channel. On the other hand, it is consumers' behaviour that induces the procyclicality of the real interest rates and hence a highly anchored expectations mitigate the real interest rate effect. Bottom line, this improvement only affects inflation rates and – like in the previous results – would be associated in higher volatility of quantities.

There are also a few scenarios in which the variance of the real wages could be reduced as compared to rational producers and consumers. The optimum one combines rationality of producers and extrapolative consumers. This, however, is highly undesirable when other variables are taken into account. Variability of the real wages acts as a mechanism of shock absorption. A scenario of lower real wage volatility can be therefore treated analogously to higher labour market rigidity, which disables deep adjustments in wages at the cost of higher output volatility.

The same applies to lower inflation volatility in some scenarios (both under full homogeneity of expectation types and when they are differentiated between groups of agents). Inflation differentials within the euro area are sometimes interpreted as an artefact of efficient adjustment mechanisms (Honohan and Lane, 2003; European Commission, 2006). When all groups (or a subgroup of agents) represent e.g. strongly anchored expectations, this can very efficiently stabilize inflation, but disable country-level adjustment after asymmetric shocks.

Tables 9-12 also reveal that the type of producers' expectations affects macroeconomic volatility of a small open monetary union member country to a larger extent than consumers' ones. In particular, variance of the system's variables takes extremely high values when producers' expectations are static, partly extrapolative or – to a lesser extent – adaptive. Also, it is the producers' dimension along which the pattern of instability is determined in a more remarkable way.

Table 9: Variance of tradable output under different expectation types of producers vs consumers (rescaled: A vs A =1)

y^T

producers	A	B	C1	C2	D1	D2	E	F	G	H	I	J	K
								consumers					
(A) rational	**1.00**	1.99			1.10	1.50	2.56	1.44	1.38	1.31	1.11	1.31	1.64
(B) adaptive ($a = 0.5$)	11.55				11.61	27.44		75.42	17.19	14.12	14.98	27.96	
(C1) extrapolative ($a = 0.66$)													
(C2) extrapolative ($a = 0.33$)													
(D1) regressive ($a = 0.33$)	3.17	5.45			2.83	3.78	6.46	4.49	3.98	3.73	3.18	3.81	4.93
(D2) regressive ($a = 0.66$)	4.94	11.92			4.89	7.13	22.11	8.32	6.80	6.10	5.32	6.69	9.81
(E) static	3189.83				168.86								
(F) $0.5A + 0.5B$	1.97	4.94			2.17	3.26	7.43	3.22	2.70	2.47	2.25	2.80	3.92
(G) $0.5A + 0.5C1$	141.70							143.11	94.98	93.97		704.48	157.23
(H) $0.5A + 0.5C2$	1074.03							1004.18	625.98	587.91			1145.27
(I) $0.5A + 0.5D1$	1.82	3.56			1.88	2.58	4.45	2.68	2.40	2.24	1.96	2.35	3.05
(J) $0.5A + 0.5D2$	2.00	4.72			2.18	3.22	6.77	3.22	2.78	2.53	2.24	2.79	3.86
(K) $0.5A + 0.5E$	2.70	14.52			3.32	6.43	38.59	6.04	4.41	3.68	3.44	4.89	8.54

y^{NT}

producers	A	B	C1	C2	D1	D2	E	F	G	H	I	J	K
								consumers					
(A) rational	**1.00**	1.97			1.33	1.46	2.87	1.20	1.06	1.03	1.21	1.22	1.28
(B) adaptive ($a = 0.5$)	4.41				7.15	19.45		39.73	6.35	5.06	8.47	15.18	
(C1) extrapolative ($a = 0.66$)													
(C2) extrapolative ($a = 0.33$)													
(D1) regressive ($a = 0.66$)	1.95	3.25			2.65	2.84	3.80	2.27	2.04	2.00	2.41	2.39	2.33
(D2) regressive ($a = 0.33$)	3.09	15.90			5.72	7.28	40.96	5.02	3.49	3.29	5.03	5.18	5.62
(E) static	615.59				69.44								
(F) $0.5A + 0.5B$	1.36	7.78			2.06	2.79	20.96	2.13	1.48	1.41	1.86	2.03	2.55
(G) $0.5A + 0.5C1$	38.50							38.84	24.25	24.23		220.31	43.06
(H) $0.5A + 0.5C2$	149.77							143.56	85.57	80.43			166.91
(I) $0.5A + 0.5D1$	1.64	3.62			2.32	2.63	5.08	2.03	1.71	1.67	2.13	2.14	2.14
(J) $0.5A + 0.5D2$	1.64	5.89			2.71	3.34	11.60	2.23	1.71	1.66	2.34	2.37	2.47
(K) $0.5A + 0.5E$	1.91	23.61			3.84	6.70	109.14	3.99	2.06	1.96	3.12	3.66	6.13

Table 10: Variance of consumption and real wages under different expectation types of producers *vs* consumers (rescaled: A vs A =1)

c							consumers							
producers	A	B	C1	C2	D1	D2	E	F	G	H	I	J	K
(A) rational	**1.00**	1.55			1.18	1.26	1.99	1.14	1.05	1.03	1.11	1.13	1.20
(B) adaptive ($a = 0.5$)	4.07				4.75	12.40		33.95	6.40	5.05	6.10	11.72	
(C1) extrapolative ($a = 0.66$)													
(C2) extrapolative ($a = 0.33$)													
(D1) regressive ($a = 0.33$)	1.69	2.28			2.02	2.10	2.49	1.86	1.75	1.73	1.90	1.90	1.90
(D2) regressive ($a = 0.66$)	2.72	10.44			4.28	5.23	25.37	4.14	3.10	2.92	3.91	4.09	4.66
(E) static	660.30				47.76								
(F) $0.5A + 0.5B$	1.20	4.62			1.58	1.96	11.37	1.70	1.29	1.24	1.49	1.60	1.98
(G) $0.5A + 0.5C1$	62.71							63.29	40.64	40.32		332.11	69.25
(H) $0.5A + 0.5C2$	315.55							304.83	182.85	171.26			351.66
(I) $0.5A + 0.5D1$	1.46	2.47			1.81	1.98	3.11	1.70	1.51	1.48	1.72	1.74	1.76
(J) $0.5A + 0.5D2$	1.43	3.69			2.05	2.38	6.43	1.81	1.49	1.45	1.85	1.87	1.96
(K) $0.5A + 0.5E$	1.52	14.17			2.71	4.28	58.91	2.94	1.63	1.55	2.28	2.62	4.51

rw							consumers							
producers	A	B	C1	C2	D1	D2	E	F	G	H	I	J	K
(A) rational	**1.00**	3.59			2.35	2.45	6.48	1.17	**0.76**	**0.77**	1.57	1.38	1.45
(B) adaptive ($a = 0.5$)	18.13				42.40	75.48		41.05	12.50	15.53	32.89	34.56	
(C1) extrapolative ($a = 0.66$)													
(C2) extrapolative ($a = 0.33$)													
(D1) regressive ($a = 0.66$)	1.24	3.97			3.91	3.28	6.29	1.17	**0.92**	1.03	2.24	1.61	1.38
(D2) regressive ($a = 0.33$)	3.66	26.00			13.18	13.04	79.85	2.67	2.65	3.24	7.38	4.96	2.91
(E) static	4613.51				667.92								
(F) $0.5A + 0.5B$	2.60	21.63			6.83	8.30	62.93	3.29	1.73	1.91	4.45	3.98	4.55
(G) $0.5A + 0.5C1$	41.64							51.68	20.38	21.85		368.89	79.48
(H) $0.5A + 0.5C2$	154.88							161.25	75.90	74.06			246.62
(I) $0.5A + 0.5D1$	1.40	5.99			4.12	4.00	10.92	1.55	**0.98**	1.06	2.55	2.02	1.93
(J) $0.5A + 0.5D2$	2.23	13.32			6.77	7.40	30.27	2.54	1.52	1.72	4.14	3.38	3.28
(K) $0.5A + 0.5E$	5.75	77.62			16.33	25.71	343.78	9.32	4.20	4.76	10.61	10.57	15.93

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Table 11: Variance of tradable and nontradable inflation under different expectation types of producers vs consumers (rescaled: A vs A =1)

π^T

producers	A	B	C1	C2	D1	D2	E	F	G	H	I	J	K
									consumers				
(A) rational	**1.00**	1.44			**0.61**	**0.91**	1.76	1.29	1.58	1.61	**0.83**	1.06	1.46
(B) adaptive ($a = 0.5$)	13.54				5.42	13.91		58.84	21.53	18.70	9.35	19.39	
(C1) extrapolative ($a = 0.66$)													
(C2) extrapolative ($a = 0.33$)													
(D1) regressive ($a = 0.33$)	1.29	1.74			**0.75**	1.12	2.17	1.62	1.92	1.93	1.03	1.32	1.82
(D2) regressive ($a = 0.66$)	2.81	5.34			1.72	2.91	9.80	4.29	4.14	3.87	2.35	3.23	5.14
(E) static	2192.81				81.96								
(F) $0.5A + 0.5B$	1.98	4.67			1.21	2.15	6.68	3.51	3.14	2.95	1.72	2.49	4.70
(G) $0.5A + 0.5C1$	21.59							50.67	19.39	17.08		154.57	76.08
(H) $0.5A + 0.5C2$	58.78							146.07	46.34	38.01			225.24
(I) $0.5A + 0.5D1$	1.41	2.08			**0.81**	1.26	2.59	1.86	2.13	2.13	1.13	1.48	2.15
(J) $0.5A + 0.5D2$	1.95	3.55			1.15	1.98	4.92	2.89	3.00	2.88	1.61	2.22	3.53
(K) $0.5A + 0.5E$	3.88	15.32			2.57	6.09	31.65	10.42	6.94	5.86	3.82	6.60	17.70

π^{NT}

producers	A	B	C1	C2	D1	D2	E	F	G	H	I	J	K
									consumers				
(A) rational	**1.00**	2.44			**0.34**	**0.73**	3.94	1.70	1.72	1.65	**0.64**	1.06	2.25
(B) adaptive ($a = 0.5$)	28.89				5.67	26.42		172.32	52.51	43.67	15.07	42.90	
(C1) extrapolative ($a = 0.66$)													
(C2) extrapolative ($a = 0.33$)													
(D1) regressive ($a = 0.66$)	1.93	3.96			**0.60**	1.27	5.44	2.98	2.89	2.77	1.13	1.84	3.74
(D2) regressive ($a = 0.33$)	5.18	26.23			1.88	4.92	79.41	11.48	7.97	7.06	3.62	6.50	15.86
(E) static	3086.21				83.24								
(F) $0.5A + 0.5B$	2.40	17.61			**0.90**	2.53	45.71	7.27	4.44	3.83	1.83	3.64	12.03
(G) $0.5A + 0.5C1$	64.82							159.01	59.09	53.31		502.99	248.35
(H) $0.5A + 0.5C2$	214.39							539.56	171.62	145.45			847.11
(I) $0.5A + 0.5D1$	2.01	5.29			**0.61**	1.40	8.61	3.46	3.20	3.03	1.19	2.04	4.55
(J) $0.5A + 0.5D2$	2.88	10.18			**0.94**	2.39	20.94	5.88	4.77	4.34	1.84	3.35	8.45
(K) $0.5A + 0.5E$	4.59	48.82			2.08	7.36	209.75	21.43	9.22	7.18	4.30	10.04	47.51

Table 12: Variance of external and internal terms of trade under different expectation types of producers vs consumers (rescaled: A vs A =1)

s

producers	A	B	C1	C2	D1	D2	E	F	G	H	I	J	K
(A) rational	**1.00**	2.01			1.09	1.50	2.57	1.44	1.38	1.30	1.11	1.31	1.63
(B) adaptive ($a = 0.5$)	8.93				10.00	23.26		62.73	13.26	10.78	12.58	23.24	
(C1) extrapolative ($a = 0.66$)													
(C2) extrapolative ($a = 0.33$)													
(D1) regressive ($a = 0.33$)	2.82	4.93			2.59	3.46	5.90	3.99	3.55	3.33	2.89	3.44	4.39
(D2) regressive ($a = 0.66$)	4.24	11.07			4.26	6.35	21.13	7.25	5.85	5.24	4.66	5.89	8.57
(E) static	2511.40				133.54								
(F) $0.5A + 0.5B$	1.87	4.72			2.07	3.09	7.20	2.92	2.52	2.34	2.13	2.61	3.49
(G) $0.5A + 0.5C1$	131.62							133.20	87.93	86.99		658.24	146.48
(H) $0.5A + 0.5C2$	999.24							937.24	582.00	546.40			1070.23
(I) $0.5A + 0.5D1$	1.67	3.32			1.75	2.41	4.18	2.43	2.19	2.05	1.82	2.17	2.76
(J) $0.5A + 0.5D2$	1.79	4.34			1.97	2.94	6.28	2.86	2.48	2.27	2.03	2.52	3.42
(K) $0.5A + 0.5E$	2.26	12.31			2.80	5.38	32.87	4.88	3.61	3.06	2.89	4.05	6.86

x

producers	A	B	C1	C2	D1	D2	E	F	G	H	I	J	K
(A) rational	**1.00**	4.20			1.71	2.34	7.59	1.66	1.40	1.33	1.51	1.66	1.96
(B) adaptive ($a = 0.5$)	4.70				11.39	27.84		22.40	3.93	4.84	9.64	11.47	
(C1) extrapolative ($a = 0.66$)													
(C2) extrapolative ($a = 0.33$)													
(D1) regressive ($a = 0.66$)	2.06	8.30			3.43	4.55	12.08	3.76	2.74	2.52	3.19	3.53	4.31
(D2) regressive ($a = 0.33$)	1.80	33.15			5.00	8.11	112.02	5.41	2.56	2.24	4.25	4.82	7.12
(E) static	17.94				63.21								
(F) $0.5A + 0.5B$	2.07	18.71			3.47	5.80	60.48	3.33	2.68	2.56	3.07	3.48	3.96
(G) $0.5A + 0.5C1$	74.84							92.10	54.46	52.66		408.19	104.63
(H) $0.5A + 0.5C2$	755.64							783.50	453.77	421.79			908.15
(I) $0.5A + 0.5D1$	1.42	7.79			2.80	3.85	14.45	2.49	1.88	1.78	2.40	2.57	2.94
(J) $0.5A + 0.5D2$	1.54	12.83			3.25	4.99	33.00	2.76	1.99	1.91	2.69	2.92	3.41
(K) $0.5A + 0.5E$	2.64	46.37			5.04	10.87	301.71	4.84	3.43	3.27	4.15	5.00	6.52

Note: columns A–K are headed "consumers".

5 Conclusions

In this paper, the role of expectations in the cross-country adjustment process within a monetary union is considered. We build a 2-region, 2-sector New Keynesian DSGE model of a monetary union including 4 types of asymmetric shocks: demand, tradable supply, nontradable supply and labour supply. We concentrate on the adjustment process in the small home economy, accounting for approximately 3% economic size of the monetary union. The model was calibrated so as to reflect basic structural characteristics of a median euro area economy.

Our main finding is that rational expectations minimize macroeconomic volatility in the presence of asymmetric shocks. This specification of expectations outperforms any other type in consideration: static, adaptive, regressive and extrapolative, as well as "hybrid" types. This conclusion holds for most of the economic variables. When we allow producers and consumers (also home and foreign agents) to form expectations with different mechanisms, a scenario of rational expectations in both groups generally outperforms other combinations.

The exceptions from this rule include mostly regressive (and strongly anchored) expectations in one (or both) groups and occur mainly for inflation rates in the tradable and nontradable sector, as well as real wages. In such a world, prices could be more stable at the cost of more volatile quantities. In the context of a monetary union, this can be interpreted as dampening of inflation volatility that reduces the efficiency of the cross-country adjustment process via inflation differentials. It should be emphasized that the expectations formed by producers can influence the home economy to a greater extent than the type represented by consumers. The impact of foreign agents' type of expectations is also non-negligible, albeit lower than domestic agents' ones.

Finally, it should be noted that an inherent instability – as characterized by Walters (1994) – is present in the model under extrapolative, static and adaptive expectations. Because the monetary policy targets mainly the foreign economy, the above conclusion mainly applies for domestic agents. In this context, rational expectations can be seen as an effective supplement for other adjustment mechanisms, including the competitiveness channel.

The above results stress the importance of economic education and information campaign in the process of euro adoption. If economic agents are familiar with the economic system and correctly anticipate its dynamics, they are able to smooth a large proportion of the increased macroeconomic volatility, associated with the loss of autonomous monetary policy and possibility of adjustments via the FX market. Agents' rationality can substantially support the competitiveness channel and dampens the real interest rate procyclicality inside a monetary union.

Acknowledgements

References

[1] Allsopp C., Vines D. (2010): Fiscal policy, intercountry adjustment and the real exchange rate, chapter in: Buti M., Deroose S., Gaspar V., Martins J. N. (ed.), *The euro: the first decade*, Cambridge University Press, Cambridge, str. 511-551.

[2] Andersson M., Gieseck A., Pierluigi B., Vidalis N. (2008): Wage growth dispersion across the euro area countries. Some stylised facts, *European Central Bank Occasional Paper Series*, URL http://www.ecb.int/pub/scientific/ops/date/html/index.en.html.

[3] Benigno P. (2004): Optimal monetary policy in a currency area, *Journal of International Economics*, 63(2), 293–320.

[4] Blanchard O.J., Kahn C.M. (1980): The solution of linear difference models under rational expectations, *Econometrica*, 48(5), 1305–1311.

[5] Blessing J. (2008): How costly is lasting structural heterogeneity of euro area member states? Welfare results from a two-region two-sector DSGE model, paper presented at 12th International ZEI Summer School in Bad Honnef in June 2008, URL http://www.iiw.uni-bonn.de/summerschool/2008/Blessing_ZEI08.pdf.

[6] Branch W.A., McGough B. (2009): A New Keynesian model with heterogeneous expectations, *Journal of Economic Dynamics and Control*, 33(5), 1036–1051.

[7] Brissimis S.N., Skotida I. (2008): Optimal monetary policy in the euro area in the presence of heterogeneity, *Journal of International Money and Finance*, 27(2), 209–226.

[8] Calmfors L., Johansson A. (2006): Nominal wage flexibility, wage indexation and monetary union, *The Economic Journal*, 116(506), 283–308.

[9] Calvo G. (1983): Staggered prices in a utility-maximizing framework, *Journal of Monetary Economics*, 12(3), 383–398.

[10] Chari V.V., Kehoe P.J., McGrattan E.R. (2002): Can sticky price models generate volatile and persistent real exchange rates?, *Review of Economic Studies*, 69(3), 533–563.

[11] Clarida R., Galí J., Gertler M. (1999): The science of monetary policy: a New Keynesian perspective, *Journal of Economic Literature*, 37(4), 1661–1707.

[12] Davig T., Leeper E.M. (2007): Generalizing the Taylor principle, *American Economic Review*, 97(3), 607–635.

[13] Erzeg C., Henderson D., Levin A. (2000): Optimal monetary policy with staggered wage and price contracts, *Journal of Monetary Economics*, 46(2), 281–313.

[14] European Commission (2006): The EU economy 2006 review: adjustment dynamics in the euro area – experiences and challenges, *European Economy*, 6/2006, URL `http://ec.europa.eu/economy_finance/publications/publication425_en.pdf`.

[15] European Commission (2008): EMU@10: successes and challenges after 10 years of Economic and Monetary Union, European Economy, 2/2008, URL `http://ec.europa.eu/economy_finance/emu10/emu10report_en.pdf`.

[16] European Commission (2009): Quarterly report on the euro area, 1/2009, URL `http://ec.europa.eu/economy_finance/publications/qr_euro_area/index_en.htm`.

[17] Frankel J.A., Rose A.K. (1998): The endogeneity of the optimum currency area criteria, *The Economic Journal*, 108(449), 1009–1025.

[18] Fuhrer J.C. (2000): Habit formation in consumption and its implications for monetary-policy models, *American Economic Review*, 90(3), 367–390.

[19] Galí J. (2008): *Monetary policy, inflation, and the business cycle: an introduction to the New Keynesian framework*, Princeton University Press, Princeton.

[20] Galí J., Gertler M. (1999): Inflation dynamics: a structural econometric analysis, *Journal of Monetary Economics*, 44(2), 195–222.

[21] Galí J., Gertler M., López-Salido J.D. (2001): European inflation dynamics, *European Economic Review*, 45(7), 1237–1270.

[22] Galí J., Monacelli T. (2005): Monetary policy and exchange rate volatility in a small open economy, *Review of Economic Studies*, 72(3), 707–734.

[23] Gradzewicz M., Makarski K. (2009): The macroeconomic effects of losing autonomous monetary policy after the euro adoption in Poland, *National Bank of Poland Working Papers (Materiały i Studia NBP)*, 58, URL `http://www.nbp.pl/home.aspx?f=/publikacje/materialy_i_studia/informacja.html` .

[24] Honohan P., Lane P. (2003): Divergent inflation rates in EMU, *Economic Policy*, 18(37), 357–394.

[25] Kenen P. (1969): The theory of optimum currency area: an ecclectic view, chapter in: Mundell R., Swoboda A. (ed.), *Monetary problems of the international economy*, University of Chicago Press, Chicago, pp. 41-60.

[26] Kolasa M. (2009): Structural heterogeneity or asymmetric shocks? Poland and the euro area through the lens of a two-country DSGE model, *Economic Modelling*, 26(6), 1245–1269.

[27] Koloch G. (2010): A note on aggregation of heterogeneous expectations in a New Keynesian economy, *IBS Working Paper*, 02/2010, URL `http://ibs.org.pl/site/upload/publikacje/working_papers/Note_On_Aggregation_Expectation_IBS_WP.pdf`.

[28] Lipińska A. (2008): The Maastricht convergence criteria and optimal monetary policy for the EMU accession countries, *European Central Bank Working Paper Series*, 896, URL `http://www.ecb.int/pub/scientific/wps/date/html/index.en.html`.

[29] Lombardo G. (2006): Inflation targeting rules and welfare in an asymmetric currency area, *Journal of International Economics*, 68(2), 424–442.

[30] Lucas R.E. (1976): Econometric policy evaluation: a critique, *Carnegie-Rochester Conference Series on Public Policy*, 1, 19–46.

[31] McKinnon R.I. (1963): Optimum currency areas, *American Economic Review*, 53(4), 717–725.

[32] Mongelli F.P. (2002): "New" views on the optimum currency area theory: what is EMU telling us?, *European Central Bank Working Paper Series*, 138, URL `http://www.ecb.int/pub/scientific/wps/date/html/index.en.html`.

[33] Mundell R. (1961): A theory of optimum currency areas, *American Economic Review*, 51(4), 657–665.

[34] Muth J.F. (1961): Rational expectations and the theory of price movements, *Econometrica*, 29(3), 315–335.

[35] National Bank of Poland (2009): Report on full membership of the Republic of Poland in the third stage of the EMU, `www.nbp.pl`.

[36] Pilbeam K. (2006): *International Finance*, Palgrave Macmillan, Basingstoke, New York, 3rd edition.

[37] Roubini N., Parisi-Capone E., Menegatti C. (2007): Growth differentials in the EMU: facts and considerations, *A Roubini Global Economics Service*, URL `http://www.roubini.com`.

[38] Sauer S., Sturm J.E. (2003): Using Taylor rules to understand ECB monetary policy, *German Economic Review*, 8(3), 375–398.

[39] Smets F., Wouters R. (2003): An estimated dynamic stochastic general equilibrium model of the euro area, *Journal of the European Economic Association*, 1(5), 1123–1175.

[40] Stockman A., Tesar L. (1995): Tastes in a two – country model of the business cycle: explaining international comovement, *American Economic Review*, 85(1), 168–185.

[41] Taylor J.B. (1993): Discretion versus policy rules in practice, *Carnegie-Rochester Conference Series on Public Policy*, 39(1), 195–214.

[42] Torój A. (2009a): Macroeconomic adjustment and heterogeneity in the euro area, *National Bank of Poland Working Papers (Materiały i Studia NBP)*, 54, URL `http://www.nbp.pl/home.aspx?f=/publikacje/materialy_i_studia/informacja.html` .

[43] Torój A. (2009b): Solving forward-looking models of cross-country adjustment within the euro area, *Central European Journal of Economic Modelling and Econometrics*, 1(3), 211–241.

[44] Torój A. (2010): Macroeconomic adjustment after asymmetric shocks in a heterogeneous currency union: the euro area experience 1999-2009, *mimeo*.

[45] Walters A. (1994): Walters critique, chapter in: Matthews K. (ed.), *The economics and politics of money: the selected essays of Alan Walters*, Edward Elgar Publishing, Northampton, pp. 368–370.

Appendix: structure of the model

The constant relative returns to scale household utility function takes the following form:

$$U_t\left(C_t, N_t, H_t\right) = \varepsilon_{d,t}\frac{\left(C_t - H_t\right)^{1-\sigma}}{1-\sigma} - \varepsilon_{l,t}\frac{N_t^{1+\phi}}{1+\phi} \tag{26}$$

where C_t – consumption at t, H_t – stock of consumption habits at t, N_t – hours worked at t, $\sigma > 0$ and $\phi > 0$. Consumption habits are assumed to be proportional to consumption at $t-1$:

$$H_t = hC_{t-1} \tag{27}$$

with $h \in [0; 1)$ The overall consumption index aggregates the tradable and nontradable consumption bundles:

$$C_t \equiv \left[(1-\kappa)^{\frac{1}{\delta}} C_{T,t}^{\frac{\delta-1}{\delta}} + \kappa^{\frac{1}{\delta}} C_{N,t}^{\frac{\delta-1}{\delta}}\right]^{\frac{\delta}{\delta-1}} \tag{28}$$

where $\kappa \in (0; 1)$ and $\delta > 0$.
The domestic consumption of tradables at t consists of $C_{H,t}$ and $C_{F,t}$:

$$C_{T,t} \equiv \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{29}$$

An analogous relationship holds for the foreign economy.
A single type of good is indexed with k and belongs to good variety indexed over the interval $\langle 0; 1\rangle$.
The consumption of domestic tradable goods in the home economy ($C_{H,t}$) and in the foreign one ($C_{H,t}^*$) is defined as:

$$
\begin{aligned}
C_{H,t} &\equiv \left[\left(\tfrac{1}{w}\right)^{\frac{1}{\varepsilon_T}} \int_0^1 \left(\int_0^w C_{H,t,k}^j dj\right)^{\frac{\varepsilon_T-1}{\varepsilon_T}} dk\right]^{\frac{\varepsilon_T}{\varepsilon_T-1}} \\
C_{H,t}^* &\equiv \left[\left(\tfrac{1}{w}\right)^{\frac{1}{\varepsilon_T}} \int_0^1 \left(\int_0^w C_{H,t,k}^{j*} dj\right)^{\frac{\varepsilon_T-1}{\varepsilon_T}} dk\right]^{\frac{\varepsilon_T}{\varepsilon_T-1}}
\end{aligned}
\tag{30}
$$

The parameter $\varepsilon_T > 1$ measures the elasticity of substitution between various types of goods in international trade, k indexes the variety of goods, and j – the households (integral over j reflects the difference in both economies' size). We define in an analogous way the domestic and foreign consumption of goods produced abroad, $C_{F,t}$ and $C_{F,t}^*$. For both tradable consumption baskets (i.e. H and F), we define equal elasticity of substitution between various types of goods, ε^T, both at home and abroad.
We define in an analogous way the domestic and foreign consumption of goods produced abroad, $C_{F,t}$ and $C_{F,t}^*$:

$$C_{F,t} \equiv \left[\left(\tfrac{1}{1-w} \right)^{\frac{1}{\varepsilon_T}} \int_0^1 \left(\int_w^1 C_{F,t,k}^j dj \right)^{\frac{\varepsilon_T - 1}{\varepsilon_T}} dk \right]^{\frac{\varepsilon_T}{\varepsilon_T - 1}}$$

$$C_{F,t}^* \equiv \left[\left(\tfrac{1}{1-w} \right)^{\frac{1}{\varepsilon_T}} \int_0^1 \left(\int_w^1 C_{F,t,k}^{j*} dj \right)^{\frac{\varepsilon_T - 1}{\varepsilon_T}} dk \right]^{\frac{\varepsilon_T}{\varepsilon_T - 1}} \tag{31}$$

The nontradable consumption bundles, domestic $(C_{N,t})$ and foreign $(C_{N*,t})$, are characterized in a similar fashion as:

$$C_{N,t} \equiv \left[\left(\tfrac{1}{w} \right)^{\frac{1}{\varepsilon_N}} \int_0^1 \left(\int_0^w C_{N,t,k}^j dj \right)^{\frac{\varepsilon_N - 1}{\varepsilon_N}} dk \right]^{\frac{\varepsilon_N}{\varepsilon_N - 1}}$$

$$C_{N*,t} \equiv \left[\left(\tfrac{1}{1-w} \right)^{\frac{1}{\varepsilon_{N*}}} \int_0^1 \left(\int_w^1 C_{N*,t,k}^{j*} dj \right)^{\frac{\varepsilon_{N*} - 1}{\varepsilon_{N*}}} dk \right]^{\frac{\varepsilon_{N*}}{\varepsilon_{N*} - 1}}$$

Households maximize at t the discounted flow of future utilities:

$$E_t \sum_t^\infty \beta^t U\left(C_t, N_t, H_t\right) \to \max_{C,N} \tag{32}$$

where $\beta \in (0,1)$ is households' discount factor, subject to a sequence of current and future budget constraints faced by a representative household:

$$\forall_t \quad \int_0^1 \int_0^w P_{H,t,k}^j C_{H,t,k}^j dj\, dk + \int_0^1 \int_w^1 P_{F,t,k}^j C_{F,t,k}^j dj\, dk +$$
$$+ \int_0^1 \int_0^w P_{N,t,k}^j C_{N,t,k}^j dj\, dk + E_t \left\{ Q_{t,t+1} D_{t+1} \right\} \leq D_t + W_t N_t \tag{33}$$

The right-hand side is a household's budget at t. Its income consists of payoffs of securities acquired in the previous periods (D_t), labour incomes (W_t – nominal wage for hours worked at t) and government transfers (T_t). The left-hand side of the inequality sums the consumption spendings of households (where P denotes a price of a particular consumption bundle, indexed in line with these bundles) and acquisition of securities. $Q_{t,t+1}$ is a stochastic discount factor for the payoffs at $t+1$, faced by the households.

Maximizing (32) subject to (33) leads to the following first order conditions:

  ° demand equations (home and foreign) for individual goods k produced at home:

$$C_{H,t,k} = \tfrac{1}{w} \left(\frac{P_{H,t,k}}{P_{H,t}} \right)^{-\varepsilon_T} C_{H,t}$$

$$C_{H,t,k}^* = \tfrac{1}{w} \left(\frac{P_{H,t,k}}{P_{H,t}} \right)^{-\varepsilon_T} C_{H,t}^* \tag{34}$$

° demand equations (home and foreign) for individual goods k produced abroad:

$$
\begin{aligned}
C_{F,t,k} &= \frac{1}{1-w} \left(\frac{P_{F,t,k}}{P_{F,t}} \right)^{-\varepsilon_T} C_{F,t} \\
C_{F,t,k}^* &= \frac{1}{1-w} \left(\frac{P_{F,t,k}}{P_{F,t}} \right)^{-\varepsilon_T} C_{F,t}^*
\end{aligned}
\tag{35}
$$

° demand equations (home and foreign) for individual nontradable goods:

$$
\begin{aligned}
C_{N,t,k} &= \frac{1}{w} \left(\frac{P_{N,k}}{P_N} \right)^{-\varepsilon_N} C_N \\
C_{N,t,k}^* &= \frac{1}{1-w} \left(\frac{P_{N,k}^*}{P_N^*} \right)^{-\varepsilon_N*} C_{N,t}^*
\end{aligned}
\tag{36}
$$

° demand equations (home and foreign) for domestic tradable goods:

$$
\begin{aligned}
C_{H,t} &= (1-\alpha) \left(\frac{P_{H,t}}{P_{T,t}} \right)^{-\eta} C_{T,t} \\
C_{H,t}^* &= \alpha^* \left(\frac{P_{H,t}}{P_{T,t}^*} \right)^{-\eta^*} C_{T,t}^*
\end{aligned}
\tag{37}
$$

° demand equations (home and foreign) for foreign tradable goods:

$$
\begin{aligned}
C_{F,t} &= \alpha \left(\frac{P_{F,t}}{P_{T,t}} \right)^{-\eta} C_{T,t} \\
C_{F,t}^* &= (1-\alpha^*) \left(\frac{P_{F,t}}{P_{T,t}^*} \right)^{-\eta^*} C_{T,t}^*
\end{aligned}
\tag{38}
$$

° home and foreign demand equations for all tradable goods:

$$
\begin{aligned}
C_{T,t} &= (1-\kappa) \left(\frac{P_{T,t}}{P_t} \right)^{-\delta} C_t \\
C_{T,t}^* &= (1-\kappa^*) \left(\frac{P_{T,t}^*}{P_t^*} \right)^{-\delta^*} C_t^*
\end{aligned}
\tag{39}
$$

° home and foreign demand equations for all nontradable goods:

$$
\begin{aligned}
C_{N,t} &= \kappa \left(\frac{P_{N,t}}{P_t} \right)^{-\delta} C_t \\
C_{N^*,t} &= \kappa^* \left(\frac{P_{N,t}^*}{P_t^*} \right)^{-\delta^*} C_t^*
\end{aligned}
\tag{40}
$$

° home and foreign labour supply equations:

$$
\begin{aligned}
(C_t - H_t)^\sigma \, N_t^\varphi \frac{\varepsilon_t^l}{\varepsilon_t^d} &= \frac{W_t}{P_t} \\
(C_t^* - H_t^*)^{\sigma^*} \, N_t^{*\varphi^*} \frac{\varepsilon_t^{l*}}{\varepsilon_t^{d*}} &= \frac{W_t^*}{P_t^*}
\end{aligned}
\tag{41}
$$

The respective price indices are defined in the following way:

$$P_{H,t} \equiv \left[\frac{1}{w} \int_0^1 \left(\int_0^w P_{H,t,k}^j dj \right)^{1-\varepsilon_T} dk \right]^{\frac{1}{1-\varepsilon_T}}$$

$$P_{H,t}^* \equiv \left[\frac{1}{w} \int_0^1 \left(\int_0^w P_{H,t,k}^{j*} dj \right)^{1-\varepsilon_T} dk \right]^{\frac{1}{1-\varepsilon_T}} \tag{42}$$

$$P_{F,t} \equiv \left[\frac{1}{1-w} \int_0^1 \left(\int_w^1 P_{F,t,k}^j dj \right)^{1-\varepsilon_T} dk \right]^{\frac{1}{1-\varepsilon_T}}$$

$$P_{F,t}^* \equiv \left[\frac{1}{1-w} \int_0^1 \left(\int_w^1 P_{F,t,k}^{j*} dj \right)^{1-\varepsilon_T} dk \right]^{\frac{1}{1-\varepsilon_T}} \tag{43}$$

$$P_{T,t} \equiv \left[(1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

$$P_{T,t}^* \equiv \left[(1-\alpha^*) P_{F,t}^{1-\eta} + \alpha^* P_{H,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{44}$$

$$P_{N,t} \equiv \left(\frac{1}{w} \int_0^1 \left(\int_0^w P_{N,t,k} dj \right)^{1-\varepsilon_N} dk \right)^{\frac{1}{1-\varepsilon_N}}$$

$$P_{N,t}^* \equiv \left(\frac{1}{1-w} \int_0^1 \left(\int_w^1 P_{N,t,k}^* dj \right)^{1-\varepsilon_{N*}} dk \right)^{\frac{1}{1-\varepsilon_{N*}}} \tag{45}$$

$$P_t \equiv \left[(1-\kappa) P_{T,t}^{1-\delta} + \kappa P_{N,t}^{1-\delta} \right]^{\frac{1}{1-\delta}}$$

$$P_t \equiv \left[(1-\kappa^*) P_{T,t}^{*1-\delta^*} + \kappa^* P_{N,t}^{*1-\delta^*} \right]^{\frac{1}{1-\delta^*}} \tag{46}$$

Log-linearization and differencing the formulas (44) and (46) lead to the following dependencies:

$$\pi_{T,t} = (1-\alpha) \pi_{H,t} + \alpha \pi_{F,t}$$

$$\pi_{T,t}^* = (1-\alpha^*) \pi_{F,t} + \alpha^* \pi_{H,t} \tag{47}$$

$$\pi_t = (1-\kappa) \pi_{T,t} + \kappa \pi_{N,t}$$

$$\pi_t^* = (1-\kappa^*) \pi_{T,t}^* + \alpha^* \pi_{N,t}^* \tag{48}$$

Using the above equations, we derive domestic demand functions for the domestic tradable, foreign tradable and nontradable goods:

$$C_{H,t,k} = \frac{1}{w} (1-\alpha)(1-\kappa) \left(\frac{P_{H,t,k}}{P_{H,t}} \right)^{-\varepsilon_T} \left(\frac{P_{H,t}}{P_{T,t}} \right)^{-\eta} \left(\frac{P_{T,t}}{P_t} \right)^{-\delta} C_t \tag{49}$$

$$C_{F,t,k} = \frac{1}{1-w} \alpha (1-\kappa) \left(\frac{P_{F,t,k}}{P_{F,t}} \right)^{-\varepsilon_T} \left(\frac{P_{F,t}}{P_{T,t}} \right)^{-\eta} \left(\frac{P_{T,t}}{P_t} \right)^{-\delta} C_t \tag{50}$$

$$C_{N,t,k} = \frac{1}{w}\kappa \left(\frac{P_{N,t,k}}{P_{N,t}}\right)^{-\varepsilon_N} \left(\frac{P_{N,t}}{P_t}\right)^{-\delta} C_t \tag{51}$$

Analogous equations hold for the foreign economy.
We define the stochastic discount factor as:

$$Q_{t,t+1} \equiv \frac{V_{t,t+1}}{\xi_{t,t+1}} \tag{52}$$

where $V_{t,t+1}$ is the price at t of an Arrow security, i.e. a one-period security paying 1 at $t+1$ when a specific state of nature occurs and 0 otherwise. $\xi_{t,t+1}$ is the probability that the state of nature in which 1 is paid materializes, conditional on the state of nature at t. Having the access to such a security market, households can transfer utility between periods, maximizing its discounted flow (see Galí and Monacelli, 2005). The optimality of decisions requires that the marginal loss in utility due to buying the security at t instead of allocating this money to consumption must equal the discounted payoff at $t+1$, also expressed in terms of marginal growth of future utility:

$$\frac{V_{t,t+1}}{P_t}\varepsilon_{d,t}(C_t - H_t)^{-\sigma} = \xi_{t,t+1}\beta\varepsilon_{d,t+1}(C_{t+1} - H_{t+1})^{-\sigma}\frac{1}{P_{t+1}} \tag{53}$$

whereby C_{t+1} and P_{t+1} in the above equation should be interpreted as conditional expected values given the state of nature when the payoff is nonzero.
Applying the definition of $Q_{t,t+1}$ (52) and (27), the equation (53) can be written as:

$$\beta\frac{\varepsilon_{d,t+1}}{\varepsilon_{d,t}}\left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right) = Q_{t,t+1} \tag{54}$$

We calculate the conditional expected value of both sides, which – along with $\Im_t \equiv E_t(Q_{t,t+1})$ – leads to the Euler equation for consumption:

$$\Im_t = \beta E_t\left[\frac{\varepsilon_{d,t+1}}{\varepsilon_{d,t}}\left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right] \tag{55}$$

Log-linearization of (55) around the steady state allows us to write the following dependence:

$$\begin{aligned}
c_t - hc_{t-1} &= E_t(c_{t+1} - hc_t) - \tfrac{1-h}{\sigma}[i_t - (E_tp_{t+1} - p_t) + ln\beta] + \\
&\quad + \tfrac{1-h}{\sigma}(\varepsilon_{d,t} - E_t\varepsilon_{d,t+1})
\end{aligned} \tag{56}$$

where lowercase variables are percentage deviations from the steady state for their uppercase counterparts. After basic simplifications, we obtain (see Smets and Wouters, 2003):

$$c_t \quad = \quad \frac{h}{1+h}c_{t-1} + \frac{1}{1+h}E_t c_{t+1} - \frac{1-h}{(1+h)\sigma}\left(i_t - E_t\pi_{t+1} - \rho\right) +$$
$$+\frac{1-h}{(1+h)\sigma}\left(\varepsilon_{d,t} - E_t\varepsilon_{d,t+1}\right) \tag{57}$$

where $i_t \equiv -ln\Im_t$ denotes short-term nominal interest rate at t, $E_t\pi_{t+1} = E_t p_{t+1} - p_t$ – expected domestic consumer price growth, $\rho = -ln\beta$ – natural interest rate corresponding to the households' discount factor β.

Define bilateral terms of trade between the home and foreign economy as:

$$S_t \equiv \frac{P_{H,t}}{P_{F,t}} \tag{58}$$

Log-linearizing (58) around a symmetric steady state $S_t = 1$ – the law of one price in the tradable sector – leads to the following relationship:

$$s_t = p_{H,t} - p_{F,t} \tag{59}$$

Also, define internal terms of trade as price ratio between tradables and nontradables:

$$X_t \equiv \frac{P_{T,t}}{P_{N,t}} \tag{60}$$

An analogous approximation allows us to write:

$$x_t = p_{T,t} - p_{N,t} \tag{61}$$

Using (58) and (48) we can write:

$$p_{T,t} = p_{H,t} - \alpha s_t \tag{62}$$

$$p_t = p_{T,t} - \kappa x_t = p_{N,t} + (1 - \kappa)\, x_t \tag{63}$$

The real exchange rate Q_t (q_t for log-deviation from the steady state) versus the rest of the monetary union takes the form:

$$q_t = p_t - p_t^* = (1 - \alpha - \alpha^*)\, s_t - \kappa x_t + \kappa^* x_t^* \tag{64}$$

Real exchange rate Q_t (q_t in log-deviations from the steady state) appreciation is then linked to the appreciation of external *terms of trade*, depreciation of domestic internal terms of trade (defined as in (61)) and appreciation of foreign internal terms of trade.

Under complete markets, equation (53) holds for both home and foreign economy (see Galí and Monacelli, 2005 for derivation of a more general version):

$$\frac{V_{t,t+1}}{P_t^*}\varepsilon_{d,t}^* \left(C_t^* - H_t^*\right)^{-\sigma} = \xi_{t,t+1}\beta^*\varepsilon_{d,t+1}^* \left(C_{t+1}^* - H_{t+1}^*\right)^{-\sigma}\frac{1}{P_{t+1}^*} \tag{65}$$

An equation analogous to (66), derived from (65), with a common stochastic discount factor, can be written:

$$\beta^* \frac{\varepsilon^*_{d,t+1}}{\varepsilon^*_{d,t}} \left(\frac{C^*_{t+1} - h^* C^*_t}{C^*_t - h^* C^*_{t-1}} \right)^{-\sigma} \left(\frac{P^*_t}{P^*_{t+1}} \right) = Q_{t,t+1} \tag{66}$$

Combining (54) and (66), we obtain:

$$\varepsilon_{d,t} \left(C_t - h C_{t-1} \right)^{-\sigma} = \vartheta^* \varepsilon^*_{d,t} \left(C^*_t - h^* C^*_{t-1} \right)^{-\sigma^*} Q_t \tag{67}$$

Following Galí and Monacelli (2005) we assume that $\vartheta^* = \vartheta = 1$. This does not affect the generality, except for restricting the initial conditions on the stock of net foreign assets and states of nature. Log-linearizing equation (67) around a steady-state allows to derive a relation between home and foreign consumption and the real exchange rate (see also Chari, Kehoe, McGrattan 2002):

$$\frac{\sigma}{1-h} \left(c_t - h c_{t-1} \right) - \varepsilon_{d,t} = \frac{\sigma^*}{1-h^*} \left(c^*_t - h^* c^*_{t-1} \right) - \varepsilon^*_{d,t} - q_t \tag{68}$$

The producers of variety k in the tradable or nontradable bundle face the following production function (see Galí, 2008):

$$Y^H_{t,k} = A^H_t N^H_{t,k} \varepsilon^H_t \tag{69}$$

$$Y^N_{t,k} = A^N_t N^N_{t,k} \varepsilon^N_t \tag{70}$$

whereby $ln A^H_t \equiv a^H_t$ is an exogenous technological process (analogously for the non-tradable sector N). Following Clarida, Galí, Gertler (1999), we assume away the price deviations of individual varieties within a sector as of second-order importance in the proximity of the steady state. This allows us to integrate the formulas (69) and (70) into sectoral production functions with supply shocks denoted ε^H_t and ε^N_t respectively (henceforth as recycling notation for the logs).

The real marginal cost (as log-deviation from the steady-state) is calculated as a difference between the wage level in the region (w_t) and the sectoral producer price log-level plus the log of marginal labour product (mpn):

$$\begin{aligned} mc^H_t &\equiv w_t - p^H_t - mpn^H_t \\ mc^N_t &\equiv w_t - p^N_t - mpn^N_t \end{aligned} \tag{71}$$

The real marginal product is equal across producers in a given sector. After substituting into (71) the derivatives of both functions with respect to N_t, we obtain:

$$mc^H_t = w_t - p_{H,t} - \left(a^H_t + \varepsilon^H_t \right) \tag{72}$$

$$mc_t^N = w_t - p_{N,t} - \left(a_t^N + \varepsilon_t^N\right) \tag{73}$$

Using equations (62)-(63) and the labour supply equation (41) leads to:

$$
\begin{aligned}
mc_t^H &= (w_t - p_t) + (p_t - p_{T,t}) + (p_{T,t} - p_{H,t}) - \left(a_t^H + \varepsilon_t^H\right) = \\
&= (w_t - p_t) - \alpha s_t - \kappa x_t - \left(a_t^H + \varepsilon_t^H\right)
\end{aligned} \tag{74}
$$

$$
\begin{aligned}
mc_t^N &= (w_t - p_t) + (p_t - p_{N,t}) - \left(a_t^N + \varepsilon_t^N\right) = \\
&= (w_t - p_t) + (1 - \kappa)\, x_t - \left(a_t^N + \varepsilon_t^N\right)
\end{aligned} \tag{75}
$$

As Galí and Gertler (1999) we assume that a fraction $1 - \theta$ of producers are able to change their price in t in each sector, which implies the following dependence between the price levels at $t - 1$ and t:

$$
\begin{aligned}
p_t^H &= \theta^H p_{t-1}^H + \left(1 - \theta^H\right) \bar{p}_t^H \\
p_t^N &= \theta^N p_{t-1}^N + \left(1 - \theta^N\right) \bar{p}_t^N
\end{aligned} \tag{76}
$$

where $\bar{p}_t^H$ and $\bar{p}_t^N$ denote the prices set newly at t by the $1 - \theta$ fraction of producers. Among the producers who reoptimize prices there is a fraction of $1 - \omega$ producers reoptimizing in an anticipatory manner as in Calvo (1983). They maximize the discounted flow of future profits, using all information available at the time of decision and taking into account future constraints. The rest of producers (ω) reset their prices, according to past price dynamics:

$$
\begin{aligned}
\bar{p}_t^H &= \omega^H p_{b,t}^H + \left(1 - \omega^H\right) p_{f,t}^H \\
\bar{p}_t^N &= \omega^N p_{b,t}^N + \left(1 - \omega^N\right) p_{f,t}^N
\end{aligned} \tag{77}
$$

Following Galí and Gertler (1999), prices set by the latter group of producers are modelled as reoptimized prices from the previous period, indexed to past inflation:

$$
\begin{aligned}
p_{b,t}^H &= \bar{p}_{t-1}^H + \pi_{t-1}^H \\
p_{b,t}^N &= \bar{p}_{t-1}^N + \pi_{t-1}^N
\end{aligned} \tag{78}
$$

One can show (see Galí and Gertler, 1999; Galí, Gertler, López-Salido 2001; Galí, 2008 for details) that the reoptimized prices satisfy the following conditions:

$$p_{f,t}^H = \mu^H + \left(1 - \beta\theta^H\right) \sum_{s=0}^{\infty} \left(\beta\theta^H\right)^s E_t \left(mc_{t+s}^H + p_{H,t+k}\right) \tag{79}$$

$$p_{f,t}^N = \mu^N + \left(1 - \beta\theta^N\right) \sum_{s=0}^{\infty} \left(\beta\theta^N\right)^s E_t \left(mc_{t+s}^N + p_{N,t+k}\right) \tag{80}$$

where $\mu^T \equiv -ln\frac{\varepsilon^T}{\varepsilon^T-1}$ and $\mu^N \equiv -ln\frac{\varepsilon^N}{\varepsilon^N-1}$ are log-markups in the steady state (or markups in an economy without price rigidities), mc_t – real marginal cost at t.

Combined relationships (76)-(80) lead to the following hybrid Phillips curves in both domestic sectors:

$$
\begin{aligned}
\pi_t^H &= \frac{\omega^H}{\theta^H + \omega^H \left[1 - \theta^H (1-\beta)\right]}\pi_{t-1}^H + \frac{\beta\theta^H}{\theta^H + \omega^H \left[1 - \theta^H (1-\beta)\right]}E_t\pi_{t+1}^H + \\
&+ \frac{\left(1-\omega^H\right)\left(1-\theta^H\right)\left(1-\beta\theta^H\right)}{\theta^H + \omega^H \left[1 - \theta^H (1-\beta)\right]}mc_t^H
\end{aligned}
\tag{81}
$$

$$
\begin{aligned}
\pi_t^N &= \frac{\omega^N}{\theta^N + \omega^N \left[1 - \theta^N (1-\beta)\right]}\pi_{t-1}^N + \frac{\beta\theta^N}{\theta^N + \omega^N \left[1 - \theta^N (1-\beta)\right]}E_t\pi_{t+1}^N + \\
&+ \frac{\left(1-\omega^N\right)\left(1-\theta^N\right)\left(1-\beta\theta^N\right)}{\theta^N + \omega^N \left[1 - \theta^N (1-\beta)\right]}mc_t^N
\end{aligned}
\tag{82}
$$

where mc_t now denote the deviation of real marginal cost from its long-run value in the respective sector (analogously for the foreign economy).

Equilibrium on the world markets of individual goods requires equality of overall production and consumption of every variety k in the basket of domestically produced tradables:

$$
\begin{aligned}
\int_0^w Y_{H,t,k}^j dj &= \int_0^w C_{H,t,k}^j dj + \int_w^1 C_{H,t,k}^{j*} dj = \\
&= C_{H,t,k} + C_{H,t,k}^* = \\
&= \frac{1}{w}(1-\alpha)(1-\kappa)\left(\frac{P_{H,t,k}}{P_{H,t}}\right)^{-\varepsilon^T}\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\eta}\left(\frac{P_{T,t}}{P_t}\right)^{-\delta}C_t + \\
&\quad + \frac{1}{w}\alpha^*(1-\kappa^*)\left(\frac{P_{H,t,k}}{P_{H,t}}\right)^{-\varepsilon^T}\left(\frac{P_{H,t}}{P_{T,t}^*}\right)^{-\eta^*}\left(\frac{P_{T,t}^*}{P_t^*}\right)^{-\delta^*}C_t^* = \\
&= \frac{1}{w}\left(\frac{P_{H,t,k}}{P_{H,t}}\right)^{-\varepsilon^T}\left[(1-\alpha)(1-\kappa)\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\eta}\left(\frac{P_{T,t}}{P_t}\right)^{-\delta}C_t\right] \\
&\quad + \frac{1}{w}\left(\frac{P_{H,t,k}}{P_{H,t}}\right)^{-\varepsilon^T}\left[\alpha^*(1-\kappa^*)\left(\frac{P_{H,t}}{P_{T,t}^*}\right)^{-\eta^*}\left(\frac{P_{T,t}^*}{P_t^*}\right)^{-\delta^*}C_t^*\right]
\end{aligned}
\tag{83}
$$

Plugging the above expression into the definition of aggregate domestic tradable product,

$$
Y_t^H \equiv \left[\int_0^1 \left(\int_0^w Y_{H,t,k}^j dj\right)^{\frac{\varepsilon^T-1}{\varepsilon^T}} dk\right]^{\frac{\varepsilon^T}{\varepsilon^T-1}}
\tag{84}
$$

yields:

$$
\begin{aligned}
Y_t^H &= (1-\alpha)(1-\kappa)\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\eta}\left(\frac{P_{T,t}}{P_t}\right)^{-\delta}C_t+ \\
&\quad +\alpha^*(1-\kappa^*)\left(\frac{P_{H,t}}{P_{T,t}^*}\right)^{-\eta^*}\left(\frac{P_{T,t}^*}{P_t^*}\right)^{-\delta^*}C_t^* = \\
&= (1-\alpha)(1-\kappa)S_t^{-\alpha\eta}X_t^{-\kappa\delta}C_t+\alpha^*(1-\kappa^*)S_t^{-(1-\alpha^*)\eta^*}X_t^{*-\kappa^*\delta^*}C_t^*
\end{aligned}
\tag{85}
$$

An analogous expression can be written for the sector of foreign tradables (F):

$$
\begin{aligned}
Y_t^F &= \alpha(1-\kappa)\left(\frac{P_{F,t}}{P_{T,t}}\right)^{-\eta}\left(\frac{P_{T,t}}{P_t}\right)^{-\delta}C_t+ \\
&\quad +(1-\alpha^*)(1-\kappa^*)\left(\frac{P_{F,t}}{P_{T,t}^*}\right)^{-\eta^*}\left(\frac{P_{T,t}^*}{P_t^*}\right)^{-\delta^*}C_t^* = \\
&= \alpha(1-\kappa)S_t^{-(1-\alpha)\eta}X_t^{-\kappa\delta}C_t+(1-\alpha^*)(1-\kappa^*)S_t^{-\alpha^*\eta^*}X_t^{*-\kappa^*\delta^*}C_t^*
\end{aligned}
\tag{86}
$$

Log-linearizing around the steady-state, in which ratio of consumption levels in both economies is proportional to their relative size ($\frac{C}{C^*} = \frac{w}{1-w}$) leads to the following conditions:

$$
\begin{aligned}
y_t^H &= \tilde{w}c_t+(1-\tilde{w})c_t^*-[\tilde{w}\alpha\eta+(1-\tilde{w})(1-\alpha^*)\eta^*]s_t+ \\
&\quad -\tilde{w}\kappa\delta x_t-(1-\tilde{w})\kappa^*\delta^*x_t^*
\end{aligned}
\tag{87}
$$

$$
\begin{aligned}
y_t^{F*} &= \tilde{w}^*c_t+(1-\tilde{w}^*)c_t^*+[\tilde{w}^*(1-\alpha)\eta+(1-\tilde{w}^*)\alpha^*\eta^*]s_t+ \\
&\quad -\tilde{w}^*\kappa\delta x_t-(1-\tilde{w}^*)\kappa^*\delta^*x_t^*
\end{aligned}
\tag{88}
$$

whereby:

$$
\begin{aligned}
\tilde{w} &= \frac{w(1-\alpha)(1-\kappa)}{w(1-\alpha)(1-\kappa)+(1-w)\alpha^*(1-\kappa^*)} \\
\tilde{w}^* &= \frac{w\alpha(1-\kappa)}{w\alpha(1-\kappa)+(1-w)(1-\alpha^*)(1-\kappa^*)}
\end{aligned}
\tag{89}
$$

Market clearing conditions for the nontradable sector can be written using (36) as:

$$
\begin{aligned}
Y_{N,t} = C_{N,t} = \kappa\left(\frac{P_{N,t}}{P_t}\right)^{-\delta}C_t \\
Y_{N,t}^* = C_{N,t}^* = \kappa^*\left(\frac{P_{N,t}^*}{P_t^*}\right)^{-\delta^*}C_t^*
\end{aligned}
\tag{90}
$$

Using the definition of internal terms of trade, (60), we get:

$$
\begin{aligned}
Y_{N,t} = \kappa X_t^{(1-\kappa)\delta}C_t \\
Y_{N,t}^* = \kappa^*(X_t^*)^{(1-\kappa^*)\delta^*}C_t^*
\end{aligned}
\tag{91}
$$

Log-linearizing (91) around the steady state leads to the following equilibrium conditions:

$$y_t^N = (1 - \kappa) \delta x_t + c_t \quad y_t^{N*} = (1 - \kappa^*) \delta^* x_t^* + c_t^* \tag{92}$$

In further analyses, we treat all the log-linearized variables as deviations from a "natural" state of economy, driven by the exogenous technological processes a_t^T and a_t^N and undistorted by price relations. Henceforth we drop a_t^T and a_t^N and treat y_t^T and y_t^N as output gaps in each sector.

The central bank's monetary policy is described with a Taylor (1993) rule with smoothing:

$$i_t = \rho + (1 - \gamma_\rho)(\gamma_\pi \tilde{\pi}_t + \gamma_y \tilde{y}_t) + \gamma_\rho i_{t-1} \tag{93}$$

where i_t – central bank policy rate at t, $\tilde{y}_t$ – the output gap in a currency union, $\tilde{\pi}_t$ – inflation rate in a currency union, $\gamma_\rho \in (0; 1)$ – smoothing parameter, $\gamma_\pi > 1$, $\gamma_y > 0$ – parameters of central bank's response to deviations of inflation and output from the equilibrium levels.

The output gap and inflation rate for the currency union aggregate the values for individual regions, according to their size:

$$\tilde{\pi}_t = \int_0^1 \pi_t^j dj = w\pi_t + (1 - w)\pi_t^* \tag{94}$$

$$\tilde{y}_t = \int_0^1 \tilde{y}_t^j dj = w\tilde{y}_t + (1 - w)\tilde{y}_t^* \tag{95}$$

Equation (41) would imply a perfect labour market flexibility, but we introduce nominal rigidity of nominal wages. Define mrs_t as:

$$mrs_t \equiv \frac{-\frac{\partial U(c_t, n_t)}{\partial n_t}}{\frac{\partial U(c_t, n_t)}{\partial c_t}} = \frac{\sigma}{1 - h}(c_t - hc_{t-1}) + \phi n_t + \varepsilon_t^l - \varepsilon_t^d \tag{96}$$

Sectoral production functions imply:

$$\begin{aligned} n_t &= \frac{N^N}{N}n_t^N + \frac{N^H}{N}n_t^H = \frac{\frac{Y^N}{A^N}}{\frac{Y^N}{A^N} + \frac{Y^T}{A^T}}n_t^N + \frac{\frac{Y^T}{A^T}}{\frac{Y^N}{A^N} + \frac{Y^T}{A^T}}n_t^H \approx \\ &\approx \kappa n_t^N + (1 - \kappa)n_t^H = \kappa y_t^N + (1 - \kappa)y_t^H - \kappa a_t^N + \\ &- (1 - \kappa)a_t^H - \kappa \varepsilon_t^N - (1 - \kappa)\varepsilon^H \end{aligned} \tag{97}$$

whereby the approximation assumes a long-term technological symmetry across sectors. The above equation can be used to replace employment in equation (96) by production and current values of supply shocks.

In the short run, let nominal wages be sticky and behave according to the Calvo scheme. Under monopolistic competition in the labour market, individual domestic

and foreign households supply differentated types of labour services, N_j, with the elasticity of substitution ε_w. Total labour supply at t, N_t, can be aggregated as:

$$N_t \equiv \left[\left(\frac{1}{w} \right)^{\frac{1}{\varepsilon_w}} \left(\int_0^w N_{j,t}^{\frac{\varepsilon_w-1}{\varepsilon_w}} \, dj \right) \right]^{\frac{\varepsilon_w}{\varepsilon_w-1}} \tag{98}$$

The wage index is defined similarly as:

$$W_t \equiv \left[\frac{1}{w} \int_0^w W_{j,t}^{1-\varepsilon_w} \, dj \right]^{\frac{1}{1-\varepsilon_w}} \tag{99}$$

Only a fraction of households, $1 - \theta^w \in (0;1)$, can renegotiate their wages at every period. This fraction remains constant and households allowed to reoptimize are selected at random. In particular, the probability of being allowed to renegotiate the wage does not depend on the period elapsed since the last change. Other households partly index their their wages to past consumer inflation. Their fraction is represented by the parameter $\omega^w \in (0;1)$.

Households able to renegotiate their nominal wage maximize the present and the discounted future utilities subject to constraints resulting from expected future labour demand and the fact that the wage level might remain unchainged for a number of periods. Solving this problem leads to the following wage dynamics equation:

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \frac{(1 - \theta^w)(1 - \beta\theta^w)}{\theta^w [1 + \phi\varepsilon_w]} [mrs_t - (w_t - p_t)] - \omega^w (\beta\pi_t - \pi_{t-1}) \tag{100}$$

An analogous solution holds for the foreign economy.

The log-linearized dynamic model is composed of the Euler equation for consumption (57), sectoral Phillips curves (81) and (82), wage equation (100), real marginal cost definitions (74) and (75) along with their foreign counterparts, equilibrium conditions (87), (88) and (92), equation of common monetary policy (93) and a set of identities defining the aggregate values for the monetary union (94) and (95), aggregate price dynamics and deflators.

The list of the random disturbances includes region-specific demand (ε_t^D), supply (ε_t^T, ε_t^N) and labour supply (ε_t^l) shocks, as well as monetary policy (ε_t^i) shock. Shocks of the same type are allowed to be correlated across regions, but are assumed to be independent across types. They also follow a first-order autoregressive process.